Finite Element Modeling of Submerged Aquaculture Net-pen Systems

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ABSTRACT

In the design of offshore aquaculture net-pens, computer based models are needed in order to evaluate and optimize design criteria prior to the construction and testing of expensive prototype net-pen systems. Two fundamental criteria that must be addressed are dynamic response and reliability. Practical computer modeling methods must be capable of handling the complex fluid/pen interactions that occur in high energy open ocean environments for a broad range of net-pen designs. The resulting dynamic response and stress and stress cycling information are essential input for design modification and lifetime predictions. In the paper, the finite element method is used to predict the three-dimensional dynamic response of submerged offshore net-pens and their associated moorings. Solid-framed and pre-stressed net-pen designs are modeled using simple structural elements. The solution to the nonlinear dynamic equilibrium equations is obtained using an incremental-iterative method based on an updated Lagrangian formulation. The resulting semi-discrete equations are integrated in time using the trapezoidal rule, and full Newton-Raphson equilibrium iteration is employed during each time step. Fluid loading is implemented using the Morison equation, along with Airy wave theory. The contributions to the element stiffness matrices that arise from the fluid loading are continually updated during the iterative process. Initial validation of the finite element results for a basic cage configuration is made by comparison to experimental data obtained from a simple cage model subjected to a constant current in a flume.

INTRODUCTION

In the development of the commercial aquaculture industry, the task of designing offshore net-pens for finfish, such as cod or haddock, is very difficult due to the complex wave and current loading to which these structures are subjected. Net-pen design has developed over the years mostly by trial and error and through the construction and testing of prototypes. Because this is very expensive and time consuming, analytical or computer based models are needed to evaluate design criteria such as reliability and dynamic response before prototypes are built.

Although considerable work has been carried out in the area of finite element modeling of cables in an ocean environment (see, for example, Haritos and He (1992), Singh and Verma (1988), and Lo and Leonard (1982)), relatively little effort has been put into the modeling of entire submerged structures along with their associated mooring systems. The goal of the present work is to incorporate the required finite element technology into a program for use by the open ocean aquaculture industry in the design and analysis of entire submerged net-pen systems. Upon the validation of this technology by comparing the numerical simulations with experimental measurements on scaled models and/or prototypes, it will serve as a powerful analysis tool to predict the dynamic response and fatigue life of such systems.

In order to demonstrate our present capability to model entire net-pen systems subjected to wave and current loading, we have created numerical simulations of three net-pen designs. The first design is a conceptual box cage design with a four-point mooring system. The second design is a box cage with a single point mooring system and an interior containment net. Finally, we present a towing simulation of Sea Station, a commercially available net-pen design developed by Ocean Spar Technologies, Bainbridge, Washington. The towing simulation was performed to predict the maximum velocity at which the system could be towed without experiencing structural failure. A simulation of Sea Station with a three point mooring system was also performed and the results are discussed.

To outline the paper, in the next section we present an effective computational procedure for analyzing flexible structures subjected to ocean waves and current. The numerical results are presented in the following section. Here, in order to validate the present finite element
methodology, we compare the finite element results to a static model test performed in a flume. In addition, we consider three net-pen designs and present the numerical simulations.

FINITE ELEMENT FORMULATION

In this section, we present an efficient finite element procedure for the transient analysis of submerged flexible structures subjected to ocean waves and current. To begin, we consider a single element with a circular cross-section as shown in Fig. 1. As shown in the figure, at time $t$ the element is in a reference configuration and occupies the domain $V^r$. At time $t + \Delta t$, the element is in a deformed configuration and occupies the domain $V^{t+\Delta t}$.

The principle of virtual work over the element in the deformed configuration is written as

$$\int_{V^{t+\Delta t}} \sigma_{ij}^{t+\Delta t} \delta u_{ij}^{t+\Delta t} dV + \int_{V^{t+\Delta t}} \rho \dot{u}^{t+\Delta t} \delta u^{t+\Delta t} dV = \int_{\partial V^{t+\Delta t}} f_{ij}^{t+\Delta t} \delta u_{ij}^{t+\Delta t} ds$$

Referring to the left hand side of Eq. (1), $\sigma_{ij}$ are the components of the Cauchy stress tensor, $u_i$ are the components of the displacement vector, $\rho$ is the mass density, $l$ is the length of the deformed element, $\delta u_i$ are the kinematically admissible virtual displacements imposed on the deformed configuration, and the superposed dot denotes differentiation with respect to time. The components $\delta u_i$ are obtained from the symmetric gradient of the virtual displacement field, i.e.,

$$\delta u_{ij}^{t+\Delta t} = \frac{1}{2} \left( \frac{\partial \delta u_{ij}^{t+\Delta t}}{\partial x^j} + \frac{\partial \delta u_{ij}^{t+\Delta t}}{\partial x^i} \right)$$

where the partial derivatives in Eq. (2) are taken with respect to the material coordinates in the deformed configuration. Throughout, the superscript $t + \Delta t$ is employed to identify the quantities that are measured in the deformed configuration, and the superscript $t$ is used to identify the quantities that are measured in the reference configuration. On the right hand side of Eq. (1), $b_i$ are the components of the body force vector which arises from gravity and buoyancy forces, and the $\int_{\partial V}$ are components of a vector representing the force per unit length acting at a point along the element arising from waves and current.

To describe the process of obtaining the discretized finite element equations, we now focus our attention on the first integral on the left hand side of the principle of virtual work statement (1). Because the quantities at time $t + \Delta t$ are unknown, it is necessary to recast the integral into an integral over the reference configuration. The integral in the reference configuration is given as

$$\int_{V^{t+\Delta t}} \sigma_{ij}^{t+\Delta t} \delta u_{ij}^{t+\Delta t} dV = \int_{V^r} S_{ij}^{t+\Delta t} \delta E_{ij}^{t+\Delta t} dV$$

where $S_{ij}$ are the components of the second Piola-Kirchhoff stress tensor referred to the reference configuration, and $\delta E_{ij}$ are the components of the variation of the Green-Lagrange strain tensor referred to the reference configuration. We note that the components of the Green-Lagrange strain tensor can be written in terms of the displacement components as follows:

$$E_{ij}^{t+\Delta t} = \frac{1}{2} \left( \frac{\partial u_i^{t+\Delta t}}{\partial x^j} + \frac{\partial u_j^{t+\Delta t}}{\partial x^i} + \frac{\partial u_i^{t} \partial u_j^{t+\Delta t}}{\partial x^i} \right)$$

where the repeated index implies summation from 1 to 3.

Next, we make the following incremental decompositions:

$$\delta E_{ij} = \delta E_{ij}^{t+\Delta t} = \delta E_{ij}^{t} + \Delta \delta E_{ij}$$

where $\Delta \delta E_{ij}$ and $\Delta \eta_{ij}$ are the increments in the linear and nonlinear parts of the of the Green-Lagrange strain. We note that the variation of the displacement field at time $t$ is zero because the displacement variations are imposed on the deformed configuration. By substituting the incremental decompositions (5) into Eq. (3) we obtain

$$\int_{V^r} S_{ij}^{t+\Delta t} \delta E_{ij}^{t+\Delta t} dV = \int_{V^r} S_{ij}^{t} \delta E_{ij}^{t} dV + \int_{V^r} S_{ij}^{t} \Delta \delta E_{ij} dV$$

All of the integrals that appear on the right hand side of Eq. (6) are linear in the displacement increments $\delta u_i$ except for integral involving the second Piola-Kirchhoff stress increments. In order to linearize this term, we make the following approximations:

$$\Delta S_{ij} = C_{ijkl}^{\Delta} \Delta \delta E_{kl}$$

$$\delta \Delta E_{ij} = \delta \delta E_{ij}$$

where $S_{ij}$ are the components of the second Piola-Kirchhoff stress tensor referred to the reference configuration, and $\delta E_{ij}$ are the components of the variation of the Green-Lagrange strain tensor referred to the reference configuration. We note that the components of the Green-Lagrange strain tensor can be written in terms of the displacement components as follows:

$$E_{ij}^{t+\Delta t} = \frac{1}{2} \left( \frac{\partial u_i^{t+\Delta t}}{\partial x^j} + \frac{\partial u_j^{t+\Delta t}}{\partial x^i} + \frac{\partial u_i^{t} \partial u_j^{t+\Delta t}}{\partial x^i} \right)$$

where the repeated index implies summation from 1 to 3.
where \( C^w_{ijkl} \) is the incremental stress-strain tensor referred to the reference configuration at time \( t \). By substituting Eq. (6) along with the approximations (7) into the principle of virtual work statement (1), we obtain the incremental form

\[
\int_0^t \frac{\partial}{\partial t} \mathbf{u}^w \, dt + \int_0^t \sigma^w \, dV + \int_0^t \sigma^w \, dV = \int_0^t f^w \, ds + \int_0^t \mathbf{b}^w \, ds
\]

We note that the integrals involving the inertia and gravity/buoyancy forces in Eq. (8) have been recast as integrals over the reference configuration in which the mass density and the body force components are reckoned per unit reference volume.

Without making any further manipulations to Eq. (8), we now proceed with the usual process of introducing the finite element shape functions and invoking the arbitrariness of the variations in the nodal displacement increments. This leads to the semi-discrete finite element equations which can be written as

\[
\begin{align*}
\mathbf{M} \Delta \mathbf{u}^w + \mathbf{K} \Delta \mathbf{u}^w &= \mathbf{R}^w + \mathbf{F}^w - \mathbf{P} \\
\end{align*}
\]

where \( \mathbf{M} \) and \( \mathbf{K} \) are the linear and nonlinear strain-displacement operator matrices, \( \sigma \) is the stress vector, and \( \mathbf{N} \) is the shape function matrix. The strain-displacement operator matrices are functions of the spatial coordinates in the reference configuration. The shape function matrix operating on the body force vector is a function of the coordinates in the deformed configuration. We note the strain-displacement operator matrices and the shape function matrix depend on the element type being used.

Equation (9) can be expressed in the form

\[
\mathbf{M} \Delta \mathbf{d}^w + \mathbf{K} \Delta \mathbf{d} = \mathbf{R}^w + \mathbf{F}^w - \mathbf{P}
\]

where \( \Delta \mathbf{d} \) is the vector containing the nodal displacement increments, \( \mathbf{M} = \mathbf{d}^{w-\Delta t} \), \( \mathbf{K} = \mathbf{s} \), \( \mathbf{F}^w \) is the equivalent nodal force vector due to gravity and buoyancy forces, and \( \mathbf{P}^w \) is the vector containing the internal nodal forces at time \( t \). The solution of Eq. (10) is not straightforward, because the equivalent force vector \( \mathbf{F}^w \) is highly deformation dependent. A simple, yet effective solution strategy is detailed in the next section. To outline the procedure, the Morison equation, modified to account for the relative motion between the structural element and the surrounding fluid, is first used to calculate the wave force per unit length acting on the element. The trapezoidal rule is then used to integrate the resulting equations in time. The process leads to a linearized system of equations that can be solved using full Newton-Raphson equilibrium iteration.

**Implementation of current and wave loading**

As discussed in the previous section, the contribution to the equivalent nodal force vector from the current and wave forces is written as

\[
\mathbf{F}^{i,w} = \int_0^t \mathbf{N}(\mathbf{x}^{i,w}) \mathbf{f}^{i,w}(s) \, ds
\]

where \( \mathbf{f}^{i,w}(s) \) is the fluid force per unit length acting at a point along the element. Letting the vector \( \mathbf{V}^{i,w} \) denote the velocity of a fluid particle at time \( t + \Delta t \), we now define the relative velocity and acceleration vectors as

\[
\begin{align*}
\mathbf{V}^{i,w} &= \mathbf{V}^{i,w} - \mathbf{u}^{i,w} \\
\dot{\mathbf{V}}^{i,w} &= \ddot{\mathbf{V}}^{i,w} - \ddot{\mathbf{u}}^{i,w}
\end{align*}
\]

where \( \mathbf{u}^{i,w} \) and \( \dot{\mathbf{u}}^{i,w} \) are the velocity and acceleration of a material point at time \( t + \Delta t \). The fluid velocity vector \( \mathbf{V}^{i,w} \), and the relative velocity and acceleration vectors \( \dot{\mathbf{V}}^{i,w} \) and \( \ddot{\mathbf{V}}^{i,w} \) can be decomposed into vectors acting perpendicular and tangent to the structural element as follows:

\[
\begin{align*}
\mathbf{V}^{i,w} &= \mathbf{V}^{i,w} + \mathbf{V}^{i,w}_n \\
\dot{\mathbf{V}}^{i,w} &= \dot{\mathbf{V}}^{i,w} + \dot{\mathbf{V}}^{i,w}_n \\
\ddot{\mathbf{V}}^{i,w} &= \ddot{\mathbf{V}}^{i,w} + \ddot{\mathbf{V}}^{i,w}_n
\end{align*}
\]

This decomposition is depicted schematically for the fluid velocity vector in Fig. 2. In the present paper, the velocity and acceleration of a fluid particle occupying position \( \mathbf{x}^{i,w} \) is obtained using Airy wave theory.

The fluid force per unit length acting on the element is calculated using the Morison equation modified to account for relative motion between the structural element and the surrounding fluid. The Morison equation is known to adequately predict the hydrodynamic force acting on a structural element with a circular cross-section whose

![Fig. 2. Decomposition of fluid velocity vector into vectors acting perpendicular and tangent to the structural element.](image-url)
diameter is small compared to the lengths of the waves encountered. The equation can be written as

\[ f = C_1 V_{re} + C_2 V_{s} + C_3 V_{s} + C_4 V_{re} \]  

(14)

where the constants \( C_1 \) to \( C_4 \) in Eq. (14) are defined as

\[ C_1 = \frac{1}{2} \rho_w D C_{dn} \left( V_{bn} \cdot V_{bn} \right)^{1/2} \]
\[ C_2 = C_{dh} \]
\[ C_3 = \rho_w A \]
\[ C_4 = \rho_w A C_m \]

(15)

Here in the definition (15), the quantities \( D \) and \( A \) are the diameter and cross-sectional area of the structural element in the deformed configuration, \( \rho_w \) is the mass density of the water, \( C_{dn} \) and \( C_{dh} \) are the normal and tangential drag coefficients, and \( C_m \) is the added mass coefficient. It should be mentioned that in general, the normal and tangential drag coefficients are functions of the Reynolds number. In the numerical calculations presented in this paper, we have employed expressions obtained by Choo and Casarella (1971) which are given in Appendix A.

We now define the unit vector \( n_{re} \) acting in the direction of the relative velocity vector, the unit vector \( n_{s} \) acting in the direction of the relative acceleration vector, and the unit vector \( n_{a} \) in the direction of the fluid acceleration vector. These unit vectors are defined as

\[ n_{re} = \frac{V_{re}}{\left( V_{re} \cdot V_{re} \right)^{1/2}} \]
\[ n_{s} = \frac{V_{s}}{\left( V_{s} \cdot V_{s} \right)^{1/2}} \]
\[ n_{a} = \frac{V}{\left( V \cdot V \right)^{1/2}} \]

(16)

We also define the unit tangent vector \( s \) acting parallel to the structural element. Having made these definitions, the fluid force per unit length can be expressed in matrix notation as

\[ f = C_1 n_{re} n_{re} V - C_2 s s V + C_3 s s V - C_4 n_{re} n_{re} \dot{u} \]

(17)

Using the finite element shape functions \( N(x'') \) to interpolate \( u \) and \( \dot{u} \) over the element, we now substitute Eq. (17) into Eq. (11) yielding the following expression for the equivalent nodal force vector:

\[ \mathbf{F}^{rev} = -m^{rev} \ddot{d}^{rev} + C^{rev} \dot{d}^{rev} + \mathbf{H}^{rev} \]

(18)

Here in Eq. (18), \( m^{rev} \) and \( C^{rev} \) are the virtual mass and damping matrices given as

\[ m^{rev} = \left( \int_0^1 C_1 N'' n_{re} n_{re} r \text{d}x'' \right)^{1/2} \]
\[ C^{rev} = \left( \int_0^1 C_1 N'' n_{re} n_{re} r \text{d}x'' + \int_0^1 C_1 N'' s s r \text{d}x'' \right)^{1/2} \]

(19)

and the vector \( \mathbf{H}^{rev} \) is given as

\[ \mathbf{H}^{rev} = \left( \int_0^1 C_1 N'' n_{re} n_{re} r V \text{d}x'' + \int_0^1 C_1 N'' s s r V \text{d}x'' + \int_0^1 C_1 N'' n_{re} n_{re} r V \text{d}x'' \right)^{1/2} \]

(20)

**Incremental solution procedure**

Having obtained an expression for the equivalent load vector \( \mathbf{F}^{rev} \) using the Morison equation along with an appropriate wave theory as described in the previous section, we now describe an incremental iterative solution procedure to solve Eq. (10) for the nodal displacements, velocities, and accelerations at time \( t + \Delta t \). To begin, we discretize Eq. (10) in time and consider discrete time increments \( n, n + 1, n + 2, \ldots \) etc. We assume that the nodal displacements, velocities, and accelerations are all known at time \( n \) and we seek to determine these quantities at time \( n + 1 \). Because Eq. (10) is an approximate equation, we need to solve it iteratively until the out-of-balance forces are within a desired convergence tolerance. The equation solved repetitively for \( i = 1,2,3,\ldots \) can be written as

\[ \mathbf{M} \ddot{d}^{n+1} + \mathbf{K}^{rev} \dot{d}^{n+1} = \mathbf{R}^{n+1} + \mathbf{F}^{n+1} - \mathbf{P}^{n+1} \]

(21)

where \( \ddot{d}^{n+1} \) is the predicted nodal acceleration vector at the end of the \( i \)th iteration, and \( \Delta t^{n+1} \) is the predicted increment in the nodal displacement vector. The tangent stiffness matrix \( \mathbf{K}^{rev} \) and the internal force vector \( \mathbf{P}^{n+1} \) are updated at the beginning of each iteration. We note that at the beginning of the process when \( i = 1 \), it is understood that \( \mathbf{K}^{rev} = \mathbf{K} \) and \( \mathbf{P}^{n+1} = \mathbf{P} \) (i.e., when \( i = 1 \), the tangent stiffness matrix and internal load vector are evaluated in the reference configuration at time \( n \)).

We now substitute the expression (18) for the equivalent nodal force vector into Eq. (21) obtaining

\[ \left( \mathbf{M} + \mathbf{m}^{n+1} \right) \ddot{d}^{n+1} + \mathbf{C}^{rev} \dot{d}^{n+1} + \mathbf{K}^{rev} \dot{d}^{n+1} = \mathbf{R}^{n+1} + \mathbf{H}^{rev} - \mathbf{P}^{n+1} \]

(22)

and employ the trapezoidal rule to integrate the equations in time. We note that the trapezoidal rule is identical to the Newmark-Beta method when the parameters \( \beta \) and \( \gamma \) are set to 0.25 and 0.5 respectively, see Newmark (1959). In the trapezoidal rule, the nodal accelerations and velocities at the end of the \( i \)th iteration are approximated as follows:
Table 1. Computational procedure for evaluating the incremental iterative Eq. (25).

1. Set up time independent mass matrix $M$.
2. Prescribe initial conditions; $d_0 = d(t = 0), \dot{d}_0 = \ddot{d}(t = 0)$,
   
   $\ddot{d}_0 = M^{-1}[R_0 + F_0 - P_0]$.
3. Begin time loop; $n = 1$ to $n_{steps}$.
4. Predict the velocity and acceleration; $\ddot{d}_{n+1} = -\dot{d}_n$,
   
   $\ddot{d}_{n+1} = -(4 / \Delta t)\ddot{d}_n - \ddot{d}_n$.
5. Begin equilibrium loop; $i = 1$ to $n_{iter}$.
6. Compute fluid velocity and acceleration vectors and associated unit vectors; $s^{*}_{n+1}, \{\mathbf{V}_n\}^{i+1}_{j=1}, \{\mathbf{V}_n\}^{i+1}_{j=1}, \{\mathbf{V}_n\}^{i+1}_{j=1}, \{\mathbf{V}_n\}^{i+1}_{j=1}, \{\mathbf{V}_n\}^{i+1}_{j=1}, \{\mathbf{V}_n\}^{i+1}_{j=1}, \{\mathbf{V}_n\}^{i+1}_{j=1}, \{\mathbf{V}_n\}^{i+1}_{j=1}$,
   
   $\{\mathbf{M}_{n+1}\}^{i+1}_{j=1}, \{\mathbf{M}_{n+1}\}^{i+1}_{j=1}, \{\mathbf{M}_{n+1}\}^{i+1}_{j=1}, \{\mathbf{M}_{n+1}\}^{i+1}_{j=1}, \{\mathbf{M}_{n+1}\}^{i+1}_{j=1}, \{\mathbf{M}_{n+1}\}^{i+1}_{j=1}, \{\mathbf{M}_{n+1}\}^{i+1}_{j=1}$.
7. Compute $\{C_{rel}\}^{i+1}_{j=1}$ and $\{C_{int}\}^{i+1}_{j=1}$ according to the relations given in the Appendix.
8. Assemble the right hand side vectors and matrices:
   
   $R_{n+1} = \sum \left\{ \int_{r_j} N^d dV \right\}_{j=1}^{i+1}$
   
   $H_{n+1} = \sum \left\{ \int_{r_j} C_{ijkl} n_{k,j} n_{l,j} V dS + \int_{r_j} C_{ijkl} n_{k,j} n_{l,j} V dS + \int_{r_j} C_{ijkl} n_{k,j} n_{l,j} V dS \right\}_{j=1}^{i+1}$
   
   $m_{n+1} = \sum \left\{ \int_{r_j} C_{ijkl} n_{k,j} n_{l,j} N dS \right\}_{j=1}^{i+1}$
   
   $C_{int} = \sum \left\{ \int_{r_j} C_{ijkl} n_{k,j} N dS \right\}_{j=1}^{i+1}$
   
   $P_{n+1} = \sum \left\{ \int_{r_j} B_{ijkl} N dS \right\}_{j=1}^{i+1}$
9. Set up the right hand side out of balance force vector:
   
   $\{F_{n+1}\}^{i+1}_{j=1} = R_{n+1} + H_{n+1} - (\{M + m_{n+1}\})\ddot{d}_{n+1} + C_{int}\ddot{d}_{n+1} + P_{n+1}$.
10. Check for convergence:
   
   if $\left\| \{F_{n+1}\}^{i+1}_{j=1} - \{F_{n+1}\}^{i}_{j=1} \right\| < tol$ then,
   
   $d_{n+1} = d_n + \Delta d_n$, $\ddot{d}_{n+1} = \ddot{d}_n$, $\ddot{d}_{n+1} = \ddot{d}_n$, $n = n + 1$, go to step 4.
11. Form the effective tangent stiffness matrix:
   
   $\{K^{eff}\}^{i+1}_{j=1} = (4 / \Delta t^4)\left\{ (M + m_{n+1}) + (2 / \Delta t)C_{int} + K_{int} \right\}.
12. Solve for the nodal displacement increments, i.e.,
   
   $\{\Delta d^e_{n+1}\}^{i+1}_{j=1}$
13. Update the displacements, velocities, and accelerations;
   
   $\ddot{d}_{n+1} = (4 / \Delta t^2)\Delta d_n + \ddot{d}_{n+1}$, $\ddot{d}_{n+1} = (2 / \Delta t)\Delta d_n + \ddot{d}_{n+1}$
   
   $\ddot{d}_{n+1} = \ddot{d}_{n+1} + \Delta d_{n+1}$
14. $i = i + 1$, go to step 6.

We note that Eq. (22) is still highly nonlinear because the matrices $m_{n+1}, C_{int},$ and $H_{n+1}$ are all defined in the unknown configuration at the end of the $i$th iteration. In order to fully linearize the equations, we lag the evaluation of these matrices as follows:

$$m_{n+1} = m_{n+1}$$

$$C_{int} = C_{int}$$

$$H_{n+1} = H_{n+1}$$

and substitute the approximations (24) along with the trapezoidal rule approximations (23) into Eq. (22). We thus obtain

$$\frac{4}{\Delta t^4} \left\{ (M + m_{n+1}) + \frac{2}{\Delta t}C_{int} + K_{int} \right\} \Delta d^e_{n+1} = \left\{ R_{n+1} + H_{n+1} - \left\{ (M + m_{n+1})\ddot{d}_{n+1} + C_{int}\ddot{d}_{n+1} + P_{n+1} \right\} \right\}$$

For the sake of completeness, the computational procedure for evaluating the incremental iterative Eq. (25) is given in Table 1.

NUMERICAL RESULTS

As an initial step to validate the finite element formulation presented in the previous section, we have performed a numerical simulation of a simple box cage design and compared the numerical results with data obtained experimentally by Swift et al. (1997) for the corresponding scaled model tested in a flume. The scaled model and the flume configuration are shown in Fig. 3. The frame of the scaled model was constructed from dowels with a circular cross section, and eight pound test nylon fishing line was used for the mooring line and bridle. The scaled model was then subjected to a steady current, and the tension in the mooring line versus current velocity was recorded through the use of a load cell and data acquisition system. The attitude of the cage (the angle between the mooring line and the bottom of the flume) was also recorded versus current velocity. The corresponding finite element simulation was then performed and the numerical results were compared to the experimental data. This comparison is depicted in Fig. 4 where mooring line tension and cage attitude respectively are plotted versus current velocity. As shown in the figures, excellent agreement between the finite element results and the experimental results were obtained.

Numerical simulations of net-pen systems

In order to demonstrate our present capability to model entire net-pen systems subjected to wave and current loading, we have created
numerical simulations of three net-pen designs. The first design (the front view is shown in Fig. 5) is a conceptual box cage design with a four-point mooring system. The cage dimensions and water depth are given in the figure. The cage was subjected to a constant current along with a superposition of 3 Airy surface waves which would be representative of incoming shallow water waves where the water particle motion is predominately back and forth.

The second simulation is of a box cage design with a single point mooring system and an interior containment net. The front view of this system is shown in Fig. 6 where the cage dimensions and water depth are depicted. Again, this cage system was subjected to a constant current along with 3 Airy surface waves representing deep water waves where the water particle motion is predominantly circular.

In order to demonstrate perhaps a more realistic situation, we have performed a towing simulation of Sea Station developed by Ocean Spar Technologies (OST), Bainbridge Washington, as shown in Fig. 7. Sea Station has a central spar buoy and a circular rim fastened to the spar by pre-tensioned lines (similar to spokes supporting the rim of a wheel). The towing simulation was performed in order to determine the maximum velocity at which the system could be towed without experiencing structural failure. In the numerical simulation, Sea Station was towed without an interior containment net. In the numerical simulation, the velocity at which the cage was towed varied from 0.0 m/s to 1.0 m/s at a constant acceleration of 0.5 m/s². The tow force versus velocity is plotted in Fig. 8. As shown in the figure, the tow force versus velocity curve is nonlinear, and the tow force reaches approximately 25,000 N at a velocity of 1.0 m/s. Actual tow tests performed by OST indicate that the tow force is approximately four times greater than this when an interior containment net is present. This illustrates that the interior containment net adds considerable drag to the overall system and should be included in future simulations.

Finally, a simulation of Sea Station with a three point mooring system was performed. The cage dimensions, water depth, and mooring configuration are shown in Fig. 9. The model was subjected to a superposition of deep water surface waves in the absence of current. The response of Sea Station under the wave conditions is quite stable and the motions are gentle. The behavior predicted by the numerical simulation, closely reflects the observed behavior of Sea Station in a wave environment. All of the simulations described above will be presented as an animation during the presentation, and the results will be discussed.
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REFERENCES


APPENDIX

In this appendix, we give the relationships between the normal and tangential drag coefficients $C_{Dn}$ and $C_{Dt}$ and Reynolds number $Re_n$ obtained from Choo and Casarella (1971) as follows:

$$
C_{Dn} = \begin{cases} 
8\pi \left(1 - 0.87 s^{-1}\right) & (0 < Re_n \leq 1) \\
1.45 + 8.55 Re_n^{-0.90} & (1 < Re_n \leq 30) \\
1 + 4 Re_n^{-0.50} & (30 < Re_n \leq 10^3) 
\end{cases}
$$

$$
C_{Dt} = \pi \mu (0.55 Re_n^{1/2} + 0.084 Re_n^{3/2})
$$

where $Re_n = \frac{\rho_n D}{\mu} \left(\frac{V_n}{\rho_n} \cdot \frac{V_n}{\rho_n}\right)^{1/2}$

$$
s = -0.077215665 + \ln(8/Re_n)
$$

and $\rho_n$ and $\mu$ are the mass density and viscosity of the water.

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Fig. 5. Conceptual bin cage design subjected to constant current and shallow water waves.

Fig. 6. Box cage design with interior containment net subjected to constant current and deep water waves.
Fig. 7. Towing simulation of Sea Station.

Fig. 8. Tow force versus velocity obtained from towing simulation of Sea Station.

Fig. 9. Simulation of Sea Station with 3-point mooring configuration.