DEVELOPMENT AND APPLICATION OF A THREE-DIMENSIONAL PROBABILISTIC WIND-BORNE DEBRIS TRAJECTORY MODEL

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ABSTRACT

This research presents a probabilistic debris trajectory model adapted from current 6-degree-of-freedom (6-DoF) deterministic models, in which the *aleatoric* (inherent) uncertainty is explicitly considered in the proposed probabilistic model. While the inherent randomness in the debris flight trajectory is irreducible due to the wind turbulence, variation in wind direction, gustiness of the wind event and so forth, the proposed probabilistic model seeks to address these uncertainties through Monte Carlo simulations with the appropriate statistical distributions applied to the governing equations of motion of the debris. Verification of the probabilistic debris trajectory model is performed through an analytical and visual comparison of the simulated data to wind tunnel test data. Good agreement is observed between the simulated and the wind tunnel test debris landing locations, thus confirming the applicability of the probabilistic wind-borne debris model.

A preliminary study regarding the current wind-borne debris impact methodology has illustrated that there is a significant increase in the total kinetic energy of debris impact when the dynamic parameters of the debris trajectory, translational and rotational, are considered; therefore, the proposed probabilistic model not only provides an effective method for predicting the variation of debris trajectories in a three-dimensional (3D) space, which is imperative when performing regional building envelope impact risk, but it is also capable of providing guidance on debris impact protection.
DEDICATION

I dedicate this document to my wife, Carmen, my son, Matthew, to my sister, Breanna, and to my parents, James H. Grayson Jr. and Wanda B. Grayson. Without their patience, love and support, the journey that is life would certainly be more daunting.
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CHAPTER ONE

INTRODUCTION

Wind-borne debris created by strong wind events, particularly hurricanes, is a major source of damage to the built environment. According to Minor (1994), the envelope of a building must remain intact during strong wind events to prevent the internal pressurization of the building, which can lead to an increase in failure of buildings and a subsequent increase in the injection of additional debris into the wind stream. The cumulative damage imposed by debris may lead to breaching of the building envelope and thus cause further damage to the interior contents of buildings (Wills et al. 2002). Extensive studies of building performance by Minor (1994) concluded that wind-borne debris is a principal cause of building envelope breaches during a strong wind event.

Generally, wind-borne debris in residential areas consists of roofing materials, such as roof gravel, shingles, tiles, sheathing and framing members from the roofs of low-rise buildings (Holmes 2010; Kordi et al. 2010). However, a major source of damage to building envelopes, to both the source building and buildings located downstream of the source building has been attributed to roof sheathing panel failures during strong wind events (Visscher and Kopp 2007). Current state-of-the-art wind-borne debris flight models that seek to analyze the flight trajectory of roof sheathing panels typically employ one of two methods in the assessment of building envelope impact risk. The first method, based on the innovative research of Tachikawa (1983), utilizes simplified dimensionless equations to de-couple the basic equations of motion (e.g., Tachikawa 1988; Holmes
2004; Lin et al. 2006; Lin et al. 2007; Holmes et al. 2006; Baker 2007); however, many of the subsequent studies based on this method typically only investigate the two-dimensional (2D) motion of debris in a uniform, horizontal wind. While 2D debris flight models are easy to implement, these 2D models cannot be used to realistically represent the motion of debris during actual wind storms. The second method involves the implementation of a 6-degree of freedom (6-DoF) model to describe the debris motion in a three-dimensional (3D) space (e.g., Twisdale et al. 1996; Richards et al. 2008; Noda and Nagao 2010). However, due to a lack of aerodynamic data for most debris shapes, Twisdale et al. (1996) employed a ‘random orientation 6-degree of freedom’ (RO 6-DoF) model to account for the orientation of the debris during flight, whereas the deterministic 6-DoF model presented by Richards et al. (2008) and Noda and Nagao (2010) employed experimentally determined force and moment coefficients to calculate the appropriate orientation of the debris during flight.

The objective of this study is to adapt current 6-DoF deterministic debris trajectory models into a 6-DoF probabilistic debris trajectory model that can account for the aleatoric uncertainties (inherent randomness) that are associated with physical wind-borne debris flight. While the numerical integration necessary in a 6-DoF debris trajectory model may be more computationally intensive to analyze when compared to the simplified dimensionless equation approach, the benefits, in terms of accuracy and the information available for building envelope impact risk assessment, in the opinion of the author, justify the increased computational costs.
The remaining chapters of this document provide an in-depth discussion on the development and application of a probabilistic wind-borne debris trajectory model. Chapter Two presents a detailed literature review on previous research that has had either an implicit or explicit influence on wind-borne debris trajectory analysis. Chapter Three focuses on providing the necessary background information for a general 6-DoF deterministic wind-borne debris trajectory model that serves as the basis for the 6-DoF probabilistic wind-borne debris trajectory model. Chapter Four documents the methods that were utilized to develop and calibrate the probabilistic debris trajectory model from the deterministic model. Chapter Five provides the results from the verification study that compared the simulated data to experimentally obtained wind tunnel data. Chapter Six highlights the potential application of the proposed probabilistic debris trajectory model in the study of debris impact mechanics and building envelope impact risk assessments, and Chapter Seven presents conclusions and recommendations for further research.
CHAPTER TWO

LITERATURE REVIEW

Introduction

Wind-borne debris flight is a complex process that has been difficult to model in the past without a somewhat simplistic view of the actual forces at work. However, it has yet to be determined if these simplistic views are capable of encompassing all of the necessary information that is required to accurately represent what actually occurs during the few seconds that a piece of debris is airborne. Many studies suggest that these few seconds hold the key to unlocking the answers that we seek when it comes to hurricane disaster mitigation, as there is evidence to support that wind-borne debris may produce the majority of damage to building envelopes during strong wind events (Minor 1994; Wills et al. 2002; Holmes 2010; Kordi and Kopp 2011). However, there has been a relatively small amount of research performed in the area of wind-borne debris when compared to that of direct wind loading of a structure (Lin et al. 2007). What follows is a literature review that covers the main research studies that have taken place over the past several decades in our attempts to adequately quantify this phenomenon.

Early Wind-borne Debris Trajectory Models

Early trajectory models were born out of a necessity to assess the safety of nuclear power plants from wind-borne debris created by tornadoes. These early trajectory models
were very simplistic in nature, and reduced the wind-borne debris to a point, in many cases only considering the drag forces (e.g., McDonald et al. 1974), or a combination of constant drag and lift forces (e.g., Lee 1974) acting on the debris.

The concept of the simple trajectory model was further advanced by Simiu and Cordes (1976) when they began using wind tunnel studies to obtain the aerodynamic forces and moments acting on bluff bodies using airfoil theory. However, the validity of airfoil theory is dependent on the debris being streamlined with respect to the airflow around the body of the debris, in which the flow pattern around the body is characterized by a free-stream flow separated from the surface of the debris body only by a thin boundary layer. This is in stark contrast to the flow pattern around a bluff body, in which there is separation at the leading edge corners of the body that creates an area of high shear and vorticity known as a free shear layer. This free shear layer acts similar to the boundary layer of a streamlined body; however, the free shear layer is not attached to the surface of the body as is the case with the boundary layer (Holmes 2007). Simiu and Cordes, acknowledging that the challenge was that the aerodynamic forcing functions are not known, also reduced the missiles to a single point in the absence of aerodynamic data. Since this is only valid if the debris does not tumble during flight, Simiu and Cordes developed the concept of the average drag coefficient to account for random missile tumbling, which is essentially a weighted average of the drag coefficients based on the projected areas of the debris along the chosen coordinate axes.

Twisdale et al. 1979 developed a ‘random orientation 6-degree of freedom’ (RO 6-DoF) 3D trajectory model that included the drag, lift, and side forces (Figure 2.1)
developed by the debris during flight. However, the orientation of the debris was chosen randomly which was the extent of the probabilistic nature of the trajectory model, as the actual trajectory of the debris was a deterministic process. Comparisons of the Twisdale et al. 1979 trajectory model to that of ballistic three-degree of freedom (3-DoF) trajectory analyses suggested that simpler models may not be conservative in predicting debris trajectory ranges and impact velocities.

Redmann et al. (1976) developed a full 6-DoF deterministic three-dimensional trajectory model using wind tunnel studies to obtain experimental aerodynamic coefficients to determine the drag, lift, and side forces applied to the debris, as well as the pitch, yaw, and roll (Figure 2.2) of the debris during flight. However, due to the intensive computational requirements and the requirement that aerodynamic force and moment coefficients be known over all orientations of the debris body, this study essentially reduced the 6-DoF solution down to a 3-DoF solution for use in making engineering estimates of tornado missile speeds in nuclear plant designs. A comparison was made by Redmann et al. (1976) to the aforementioned Simiu and Cordes study that used the

Figure 2.1: Representation of drag, lift and side forces on a bluff body. Side forces are normal to the page. (Adapted from Holmes 2007)
average drag coefficients to account for the missile tumbling rather than experimental aerodynamic coefficients. This comparison illustrated that the maximum horizontal velocities of an automobile from the 6-DoF trajectory model by Redmann et al. (1976) were found to be as much as 54% lower than that reported by Simiu and Cordes, but there was no way to differentiate between which model provided the “correct” results of the analysis as none of the studies of this time period were validated with experimental data.

While these previous studies were important in identifying areas of need in relation to aerodynamic coefficients, their lack of validation by experimental data, and the fundamental differences between the wind fields of tornadoses and many other extreme wind events does not provide a direct correlation into research pertaining to debris trajectories in straight-line, synoptic winds.

![Diagram of pitch, yaw, and roll](image)

Figure 2.2: Representation of the pitch, yaw, and roll of sheet-type debris.
The Wind-borne Debris Experiments of Tachikawa

Tachikawa (1983) presented three equations based on Newton’s second law that described the motion of a generic object throughout a 2D plane within a uniform wind field:

\[ m\ddot{x} = (1/2)\rho A((v - \dot{x})^2 + \dot{y}^2)(C_D\cos \beta - C_L\sin \beta) \]  \hspace{1cm} (2.1)

\[ m\ddot{y} = mg - (1/2)\rho A((v - \dot{x})^2 + \dot{y}^2)(C_D\sin \beta + C_L\cos \beta) \]  \hspace{1cm} (2.2)

\[ I\ddot{\theta} = (1/2)\rho A[((v - \dot{x})^2 + \dot{y}^2)C_M] \]  \hspace{1cm} (2.3)

where, \( \rho \) is the air density, \( g \) is the acceleration due to gravity, \( m \) is the mass, \( I \) is the debris mass moment of inertia, \( A \) is the area of the plate, \( v \) is the wind velocity (typically designated as \( U \)), \( \ell \) is the chord length of the plate (side length), \( \beta = \tan^{-1}(\dot{y}/(v - \dot{x})) \), and \( C_D, C_L, \) and \( C_M \) are the drag, lift and moment coefficients, respectively.

Tachikawa extended this common concept of 2D debris motion by incorporating dimensionless variables into the equations of motion:

\[ \tilde{X} = K((1 - \tilde{X})^2 + \tilde{Y}^2)(C_D\cos \beta - C_L\sin \beta) \]  \hspace{1cm} (2.4)

\[ \tilde{Y} = 1 - K((1 - \tilde{X})^2 + \tilde{Y}^2)(C_D\sin \beta + C_L\cos \beta) \]  \hspace{1cm} (2.5)

\[ \tilde{\theta} = \frac{K}{L_nI_n}((1 - \tilde{X})^2 + \tilde{Y}^2)C_M \]  \hspace{1cm} (2.6)

where, \( \tilde{X} \) is the dimensionless horizontal velocity (\( X = xg/U^2 \)), \( \tilde{Y} \) is the dimensionless vertical velocity (\( Y = yg/U^2 \)), \( K \) is the ratio of the aerodynamic forces to the
gravitational forces \( K = \frac{\rho U^2 A}{2mg} \), \( L_n \) is the dimensionless chord length \( L_n = \frac{g \ell}{U^2} \) or \( 1/\text{Froude number}^2 \) and \( I_n \) is the dimensionless debris mass moment of inertia \( I_n = I / ml^2 \). While this research shed light on many aspects of wind-borne debris flight, such as providing assumptions for integrating the equations of motion, and utilizing wind tunnel experiments to identify modes of motion of flat plates in flight, the most significant information to come about from the study by Tachikawa is the concept of the \( K \) parameter, which has become so fundamental to the study of wind-borne debris trajectories that Holmes et al. (2005) have proposed that it be referred to hereafter as the \textit{Tachikawa Number}. Since then, the \( K \) parameter has been widely accepted by the wind engineering and research community as the \textit{Tachikawa Number}.

In the late 1980s, Tachikawa (1988) proposed a method for estimating the distribution range of trajectories of wind-borne debris. This research utilized free-flight tests of representative wind-borne debris within a wind tunnel to obtain knowledge of the distribution characteristics of the trajectories in order to estimate their spatial distribution, and to identify the factors that influence this distribution of trajectories, such as, wind velocity profile, ascending flow, vertical reaction force, and the scale of the missile. Tachikawa presented an easily applied method for estimating these distribution parameters based on the probability distribution of the lift coefficient \( (C_L) \); however, it was recommended within the research that further studies were needed to ensure the validity of this method.
A New Direction for Wind-borne Debris Research

The completion of Tachikawa’s seminal works in the 1980’s provided a stable foundation for further research into wind-borne debris trajectories; however, the next decade witnessed a shift away from the kinematic study of wind-borne debris trajectories to a more kinetic approach in an effort to determine the after effects of wind-borne debris upon impact. This paradigm shift was instigated by an increasing concern for the integrity of the building envelope, due in no small part to the escalating costs of damage produced by extreme wind events throughout the U.S.

McDonald (1990) conducted a significant amount of experimental and analytical research on wind-borne debris impact. While his research during this period was concerned more with tornado-propelled debris, McDonald set the standard for wind-borne debris impact research by identifying common objects that were most likely to become wind-borne debris, estimating the speeds of these debris, and then quantifying the speeds required for these debris to perforate common building materials. Results from this study concluded that typical exterior building wall configurations, except for non-masonry veneers, were incapable of resisting rod-like (e.g., common dimensional lumber materials) that exceeded approximately 22 m/s.

Hurricane Andrew in 1992 was an awakening for many, in terms of coming to the realization that wind-borne debris plays an integral part in the damage that is prevalent in extreme wind events. Minor (1994) provided a comprehensive look at the evolution of wind-borne debris impact test standards up to that point, and provided further recommendations based on these investigations. While his research did not provide
insight into a particular debris trajectory or impact model, Minor did implicitly identify areas of improvement that would need to be addressed for continued advancement in protection of the building envelope from wind-borne debris.

Renewed Interest in Wind-borne Debris Trajectories

A study prepared by Applied Research Associates, Inc. (ARA) (Twisdale et al. 1996) for an insurance company, State Farm Mutual, presented the development of a methodology for evaluating the vulnerability of building envelopes to wind-borne debris in hurricanes. Dubbed HURMIS (hurricane missile risk analysis methodology), the simulation was capable of providing missile trajectories and speeds, and impact details. This study highlighted the interdependence of wind-borne debris trajectories and impact, unlike many previous studies in which the topics were treated separately. ARA provided a summary of debris transport models, and provided the advantages and limitations of each before committing to the ‘random orientation 6-degree of freedom’ (RO 6-DoF) initially developed by Twisdale et al. (1979). The RO 6-DoF model considers drag, lift, and side forces and the tumbling of the wind-borne debris is simulated by a periodic reorienting of the debris body; however, this allows for better prediction estimates over particle trajectory models with only a minimal increase in simulation efficiency.

Wills et al. (2002) sought to simplify the model for wind-borne debris analysis by linking the aerodynamics of the debris to the damage caused when it impacts a downstream building. To facilitate this simplification, Wills et al. (2002) defined three
generic types of debris, compact, rod, and plate debris (Figure 2.3), and formulated the requirements for the generic debris to take flight as:

$$\frac{1}{2} \rho_a U^2 C_F = \rho_m l g I$$  \hspace{1cm} (2.7)

where, $\rho_a$ and $\rho_m$ are the densities of air and the material respectively, $U$ is the wind speed, $C_F$ is an average force coefficient similar to that proposed by Simiu and Cordes (1976), $l$ is the characteristic dimension of the debris particle (the thickness for plates, and the equivalent diameter for rods), $g$ is the acceleration due to gravity, and $I$ is the fixture strength integrity, which is the ratio of the wind speed required for a piece of debris to break loose from its attachment to the weight of the debris. A damage function was developed on the assumption that the amount of damage was proportional to the kinetic energy of the debris at impact:

$$D = \frac{1}{2} \rho_m l^3 u^2 = \frac{1}{2} \rho_m l^3 (JU)^2$$  \hspace{1cm} (2.8)

where, $D$ is the damage produced by the debris at impact, $u$ is the debris velocity at impact, and $J$ is the ratio of the debris speed to the wind speed, which was determined to be different for each of the three generic debris types. While an explicit range of values for $J$ were not provided by Wills et al., Holmes (2010) estimates the range for all type of debris to be from 0.4 to 0.9 at impact.
Holmes (2004) studied the trajectories of spheres in strong winds (Figure 2.4), and the effects that vertical air resistance and atmospheric turbulence has on these trajectories. The inclusion of vertical air resistance in the compact debris flight equations resulted in coupled equations for the horizontal and vertical acceleration of the debris:

\[
\frac{d^2x}{dt^2} = k(U - u_m)\sqrt{((U - u_m)^2 + v_m^2)}
\]

(2.9)

\[
\frac{d^2z}{dt^2} = k(-v_m)\sqrt{((U - u_m)^2 + v_m^2)} - g
\]

(2.10)

where, \( k = \rho_a C_D / 2 \rho_m \ell \), \( U \) is the wind speed, \( u_m \) is the debris horizontal velocity, \( v_m \) is the debris vertical velocity, \( g \) is the acceleration due to gravity, \( \rho_a \) and \( \rho_m \) are the air and the debris densities respectively, \( C_D \) is the dimensionless drag coefficient of the compact debris, and \( \ell \) is the characteristic length of the debris, which is the ratio of the volume to the frontal area of the debris for spheres.
It was concluded that the vertical air resistance does have a profound effect on the trajectories of spheres during flight resulting in increased flight times, horizontal displacements and horizontal velocities of the debris. However, atmospheric turbulence increased the variability in the horizontal displacements and velocities, but had little effect on the mean values of the debris trajectory parameters.

There have been several recent studies on plates that have expanded upon the previous work of Tachikawa (e.g., Lin 2005; Holmes et al. 2006; Lin et al. 2006). These studies investigated the aerodynamic forces and moments on plates (i.e., sheet-type debris) both numerically and empirically using data obtained from wind tunnel and full-scale testing carried out at Texas Tech University. Generally, there was good agreement between the measured and calculated trajectories when lift forces proportional to the rate of rotation (i.e., the Magnus Effect) were incorporated into the calculations. The significance of the Magnus Effect on auto-rotating plates was first reported by Tachikawa (1983), and later summarized by Holmes et al. (2006) as a bilinear function:

Figure 2.4: Forces acting on a sphere during a strong wind event, including the influence of vertical air resistance (i.e., the vertical drag)
\[ C_{Lr} = 0.42(2.5\omega / \omega_0) \quad \text{for} \quad \omega / \omega_0 < 0.2, \]  
(2.11)

\[ C_{Lr} = 0.42(0.375 + 0.625\omega / \omega_0) \quad \text{for} \quad \omega / \omega_0 \geq 0.2, \]  
(2.12)

\[ \omega_0 = \frac{0.64U}{\ell} \]  
(2.13)

where, \( \omega \) is the angular velocity of the plate, \( \omega_0 \) is the steady-state angular velocity of the plate, \( U \) is the wind speed, and \( \ell \) is the characteristic length, which is the span or chord of the plate. The Magnus Effect is essentially an additional force due to the angular velocities of the rotation experienced by these plates. While the Magnus Effect can have quite a significant effect on the lift forces of a plate, its influence is less well defined for other components, such as the drag force and the pitching moment (Holmes et al. 2006).

Lin (2005), Holmes et al. (2006), Lin et al. (2006), and Lin et al. (2007) verified the importance of the Tachikawa Number (\( K \)) through an extensive number of experimental trials, in which the horizontal velocity of wind-borne debris was determined to be highly dependent upon \( K \) (lighter debris with a larger surface area, and hence a higher value of \( K \) tend to fly farther and faster, as opposed to heavier more compact debris with a lower value of \( K \)). These experimental trials resulted in a general approximation of the dimensionless horizontal velocity (\( \overline{u} \)) as an exponential function of \( K \) and the dimensionless horizontal position (\( \overline{x} \)) for compact, rod-type, and sheet-type debris:

\[ \overline{u} \approx 1 - e^{-\sqrt{\frac{2C_{D,a}K}{\overline{x}}}} \]  
(2.14)
where, $C_{D,av}$ (see Table 2.1) is the average value of the dimensionless drag coefficient for compact and rod-type debris (Lin et al. 2007), and for sheet-type debris (Lin et al. 2006).

Table 2.1: Experimentally obtained average drag coefficients

<table>
<thead>
<tr>
<th>Debris Shape</th>
<th>Average Drag Coefficient ($C_{D,av}$)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubes</td>
<td>0.809</td>
<td>0.0203</td>
</tr>
<tr>
<td>Spheres</td>
<td>0.496</td>
<td>0.0087</td>
</tr>
<tr>
<td>Rods</td>
<td>0.801</td>
<td>0.0616</td>
</tr>
<tr>
<td>Plates</td>
<td>0.911</td>
<td>0.0814</td>
</tr>
</tbody>
</table>

Baker (2007) studied the 2D motion of compact and sheet-type debris, and proposed an alternative dimensionless method to that of Tachikawa. The dimensionless parameters identified by Baker were:

$$\Omega = \frac{Mg}{(0.5 \rho AU^2)}$$

(2.15)

$$\Delta = \frac{Ml^2}{I}$$

(2.16)

$$\Phi = \frac{0.5 \rho Al}{M}$$

(2.17)

where $M$ is the mass of the debris, $g$ is the acceleration due to gravity, $\rho$ is the density of air, $A$ is the reference debris area, $U$ is the wind speed, $I$ is the mass moment of inertia, and $l$ is the reference debris length. These groups are essentially consistent with those presented by Tachikawa (Equations (2.4) through (2.6)) albeit in a slightly different dimensionless form (e.g. $\Omega$ and $\Delta$ is simply the reciprocal of $K$ and $I_n$, respectively, and $L_n$ is equal to $\Omega \Phi$).
The complex nature of debris flight requires several simplifying assumptions to obtain closed-form solutions from the equations of motion, especially pertaining to the appropriate force and moment coefficients for sheet-type debris. Baker made the following assumptions for the aerodynamic coefficients:

\[ C_D = 0.75 \left(1 + 0.65 \sin \left( 2 \beta - \frac{\pi}{2} \right) \right) \]  
\[ C_D = 1.2 \sin(2\beta) \]  
\[ C_M = 0.2 \cos \beta \left( C_D \sin \beta + C_L \cos \beta \right) \]  
\[ C_{LA} = k_{LA} \left( \bar{\omega} / \bar{\omega}_m \right) \]  
\[ C_{MA} = k_{MA} \left( 1 - \left( \bar{\omega} / \bar{\omega}_m \right) \left( \bar{\omega} / \bar{\omega}_m \right) \right) \]

where, \( C_D \) is the drag coefficient, \( C_L \) is the lift coefficient, and \( C_M \) is the moment coefficient, \( \beta \) is the direction of the wind velocity relative to the debris principal axis, \( C_{LA} \) and \( C_{MA} \) are the auto-rotating lift and moment coefficients, respectively, \( k_{LA} \) and \( k_{MA} \) are the auto-rotating lift and moment constants taken to be 0.4 and 0.12, respectively, and \( \bar{\omega}_m \) is the maximum numerical value of the mean angular velocity taken to be 0.64. The mean value of the drag coefficient (\( \bar{C}_D \)) was obtained from a plot of Equation (2.16), and was determined to be 0.75 with a range of 0.1 to 1.4. From these assumptions, Baker provided a best fit curve for the dimensionless horizontal velocity of sheet-type debris:

\[ \bar{u} = \sqrt{2C_D}\bar{x} \]
\[
\bar{x} = \frac{xp_\rho A}{2m}
\] (2.24)

where, \(\bar{x}\) is the dimensionless horizontal position, \(p_\rho\) is the air density, \(A\) is the reference area of the sheet, and \(m\) is the mass of the sheet. Baker normalized Equation (2.23) by the theoretical asymptotic limit of horizontal debris velocity to arrive at an equation that despite the different formulation of the dimensionless parameters is very similar to Equation (2.14) from Lin et al. 2006:

\[
\bar{u} \approx (1 \pm 0.51\Omega) \left( 1 - e^{\frac{-1.2\bar{x}}{0.5}} \right)
\] (2.25)

Baker’s numerical solutions for compact debris agreed well with the predicted trajectories of Holmes (2004), however, Baker’s calculations underestimated the experiments and calculations of Wills et al. (2002) by about 12%. This was expected due to Wills et al. (2002) neglecting the vertical air resistance in their calculations. The numerical calculations for the sheet-type debris varied somewhat from the experimental data of Tachikawa (1983) at lower initial angles of inclination, but were reasonable at higher initial angles of inclination; however, Baker was not able to provide a great deal of confidence in the calculations until further experimental comparisons were made.

Karimpour and Kaye (2010) provided numerical solutions to the compact debris flight equations presented by Holmes (2004) (Equations (2.9) and (2.10)) by including a mean wind velocity profile that varied with height (Figure 2.5). The mean wind velocity profile was investigated for terrain exposures B-D from ASCE 7-05 using a logarithmic equation and a power law equation:
\[ u = \frac{u_\ast \ln \left( \frac{z}{z_0} \right)}{\kappa} \quad (2.26) \]

\[ u = u_{ref} \left( \frac{z}{z_{ref}} \right)^{\alpha} \quad (2.27) \]

where, \( u_\ast \) is the friction velocity, \( \kappa \) is the surface drag coefficient, \( z_0 \) is the surface roughness length, \( z \) is the vertical displacement, \( u_{ref} \) is the mean wind speed at \( z_{ref} \), a reference height, which is usually taken as 10m, and \( \alpha = \left( \frac{1}{ln \left( z_{ref} / z_0 \right)} \right) \).

Monte Carlo simulations were performed to establish distribution parameters for flight distance and impact kinetic energy. Results from this study determined that as \( \alpha \) in the power law equation or \( z_0 \) in the logarithmic equation increased, horizontal debris flight distances decreased, which was attributed to reduced drag on the compact debris due to the lower wind speeds near the surface of the terrain, and that the mean flight distance was sensitive to the debris diameter, but not \( z_0 \). Overall, comparisons of the mean wind velocity profile predicted using the logarithmic and power law formulations provided values less than that predicted using a uniform mean wind velocity profile.
Karimpour and Kaye (2011) further investigated compact debris flight through Monte Carlo simulations that modeled the flight of a single spherical particle driven by turbulent wind with velocity fluctuations common to the atmospheric boundary layer. Variability of the particle diameter was incorporated into the simulations through a probability distribution function. This study determined that the inclusion of variability in particle diameter, and the introduction of horizontal and vertical turbulence intensities into the wind profile leads to larger mean values of the debris flight distance and impact kinetic energy compared to deterministic models.

**Current Deterministic Debris Trajectory Models**

Richards et al. (2008) and Noda and Nagao (2010) have presented deterministic 6-DoF debris trajectory models using numerical methods to solve for the equations of motion.
based on Newton’s second law. Wind tunnel test data is utilized for the aerodynamic force and moment coefficients on plate debris during flight. These deterministic debris trajectory models form the template for the probabilistic debris trajectory model developed in this report; therefore, a detailed description of these models is provided in Chapter Three of this report.

Richards et al. (2008) performed wind tunnel tests to study the 3D motion of rectangular plates and long rods during flight. These studies revealed that the aerodynamic normal force coefficient (Figure 2.6) of the test specimens was dependent on the angle of attack and the tilt angle of the plate (Figure 2.7). Richards et al. (2008) also observed plate rotation phenomena similar to Tachikawa (1983) and Baker (2007), in which moments and the mean drag force acting on the plate were modified by rotation.

Richards et al. (2008) noted that the calculated scatter of the plate debris were similar to the scatter reported by Tachikawa (1988), in that the scatter distribution range was circular with the diameter increasing with an increase in plate side length aspect ratio. Results from this study concluded that there was significant lateral movement of the debris within the 3D trajectory model, and that given enough flight time the horizontal velocity of the debris was greater than 90% and in some cases 100% of the wind speed, which was attributed to vertical momentum being converted to horizontal momentum by the forces on the plate, and the autorotation of the plate.
Figure 2.6: Normal force coefficients as a function of the angle of attack and the tilt angle for a plate with side length ratio = 2.

Figure 2.7: Definition of the flow angles with respect to the debris principal axes.
Noda and Nagao (2010) performed a wind tunnel study to investigate the 3D motion of flat plates exclusively. This investigation led to the development of a 6-DoF deterministic debris trajectory model, in which the aerodynamic force coefficients were determined experimentally using a six component load balance within a wind tunnel, similar to Richards et al. (2008). The plates were tested through a range of angles that defined the horizontal and vertical angles of attack, and the calculated trajectories were utilized to determine the effects of the Tachikawa Number ($K$), and the plate aspect ratio on the trajectories of the debris. Results from this study concluded that the flight distance of the debris increased proportionally with $K$, thereby controlling the overall trajectory of the debris in all cases; however, the extension region of the debris (i.e., scatter of the debris) was affected by $K$, the aspect ratio, and the wind speed.

**Current Research on Plate Debris**

Visscher and Kopp (2007), Kordi et al. (2010), and Kordi and Kopp (2011) conducted research at the University of Western Ontario Boundary Layer Wind Tunnel Laboratory investigating the trajectories of roof sheathing panels under high winds, and the effects that the initial wind angle and initial conditions impose on these plate trajectories. The data obtained from this research forms the basis for the calibration and verification of the probabilistic debris trajectory model developed in this report; therefore, a brief summary of these studies are presented here with a more detailed description provided in Chapter Five of this report.
Visscher and Kopp (2007) utilized a 1:20 scale failure model to evaluate the flight of roof sheathing panels. High speed digital equipment was used to capture the trajectory of the plate debris during flight, thus providing insight into the modes of flight of the panels, namely, translational and auto-rotational, with combinations of these two modes evident at times. Results from this study observed the translational mode of flight 75% of the time, and the auto-rotational mode 25%, with the auto-rotational mode providing the highest debris speeds over the longest distances and greatest variability in scatter.

Kordi et al. (2010) extended the work of Visscher and Kopp (2007) by examining the effect of the wind direction on the flight trajectories of roof sheathing panels. This was accomplished by rotating the same experimental scale model setup from Visscher and Kopp (2007) through various angles. Results from this study concluded that the local effects of the flow field along the roof of a low-rise structure has a dramatic effect on the flight trajectories of roof sheathing panels; however, the assumption of using a uniform, smooth wind appeared to provide credible results in determining panel speeds, which are imperative for building envelope impact analyses. This study also identified more modes of flight in addition to the modes of flight observed by Visscher and Kopp (2007), namely, 3D spinning, falling and a no flight condition after failure, in addition to the translational and auto-rotational modes of flight identified previously.

Kordi and Kopp (2011) investigated the effects of the initial conditions on the flight of wind-borne debris plates, in this case, roof shingles and tiles. The experimental setup remained the same as with their previous wind-borne debris research. Several limitations were identified in this study as a result of using a scaled failure model that is generally
larger than the majority of scaled models that are utilized in wind tunnel experiments. This causes the integral scales of the flow to be too small compared to that of full-scale tests. Another limitation is that the failure model utilized assumes that failure occurs from wind-induced pressures overcoming the hold down force of the component being tested. This assumption is not entirely true for roof shingles or tiles; however, the failure mechanism is assumed not to influence the debris flight trajectories in this study. Results from this study concluded that debris failing in the highest wind speeds did not always fly the farthest, and generally the debris that travels the highest also travels the farthest and the fastest. In addition, shingles that took flight exhibited speeds in the range of 40-120%, and tiles in the range of 30-60% of the mean roof height gust speed at failure.
CHAPTER THREE

DETERMINISTIC DEBRIS TRAJECTORY MODEL

There are several studies presented in current research literature that illustrate practical methods to implement a deterministic 6-DoF debris trajectory model (e.g., Richards et al. 2005; Richards et al. 2008; and Noda and Nagao 2010). The following sections present a general deterministic debris trajectory model that utilizes the assumptions and data from current research as noted. This is done to adequately illustrate the process followed in the adaptation of a deterministic debris trajectory model into a probabilistic debris trajectory model.

Deterministic Model Coordinate System

In order to adequately develop a full 6-DoF debris trajectory model, it is necessary to define the appropriate coordinate systems necessary to track all aspects of the debris trajectory (i.e., translational and rotational motion). The coordinate systems used to define this motion are the earth fixed axes (i.e., $X_e$, $Y_e$, $Z_e$), in which the center of gravity of the debris is defined by the position vector $X = [x, y, z]$ and the velocity vector $V = [V_x, V_y, V_z]$. The global earth translating, non-rotating axes (i.e., $X_G$, $Y_G$, and $Z_G$), which remain congruent to the earth’s fixed axes while moving with the debris, and the debris principal axes (i.e., $X_P$, $Y_P$, and $Z_P$), which coincide with the dimensions of the debris (i.e., $l_X$, $l_Y$, and $l_Z$) such that $l_X \leq l_Y \leq l_Z$. 

26
An understanding of the orientation of the debris principal axes (i.e., $X_P$, $Y_P$, and $Z_P$) with respect to the global translating, non-rotating axes (i.e., $X_G$, $Y_G$, and $Z_G$) is necessary for the transition of a deterministic trajectory model to a probabilistic trajectory model. As illustrated in Figure 1, the general orientation of the debris principal axes in relation to the global translating, non-rotating axes is defined using the “pitch-yaw-roll” convention of the Tait-Bryan angles ($\theta_Z$, $\theta_Y$, $\theta_X$), which leads to the following transformation equation:

$$
\begin{bmatrix}
X_P \\
Y_P \\
Z_P
\end{bmatrix}
= T(\theta_X, \theta_Y, \theta_Z) X_G = \left[ R_X(\theta_X) \right]^T \left[ R_Y(\theta_Y) \right]^T \left[ R_Z(\theta_Z) \right]^T
\begin{bmatrix}
X_G \\
Y_G \\
Z_G
\end{bmatrix}
$$

where $T$ is the global transformation matrix which relates the translating non-rotating axes to the debris principal axes, $X_G$ is the global translating, non-rotating axes vector, and $R_X$, $R_Y$, and $R_Z$ are the rotation matrices as follows:

$$
R_X(\theta_X) =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta_X & -\sin \theta_X \\
0 & \sin \theta_X & \cos \theta_X
\end{bmatrix}
$$

$$
R_Y(\theta_Y) =
\begin{bmatrix}
\cos \theta_Y & 0 & \sin \theta_Y \\
0 & 1 & 0 \\
-\sin \theta_Y & 0 & \cos \theta_Y
\end{bmatrix}
$$

$$
R_Z(\theta_Z) =
\begin{bmatrix}
\cos \theta_Z & -\sin \theta_Z & 0 \\
\sin \theta_Z & \cos \theta_Z & 0 \\
0 & 0 & 1
\end{bmatrix}
$$
In order to define counterclockwise rotation positive with respect to the debris principal axes, the transpose of the rotations matrices is taken since the rotation of the coordinate system is actually clockwise to a unit vector representing the global translating, non-rotating axes. Therefore, the global transformation matrix from Equation (3.1) becomes:

\[
T = \begin{bmatrix}
\cos \theta_x \cos \theta_y & \cos \theta_y \sin \theta_z & -\sin \theta_x \\
\sin \theta_x \sin \theta_z - \cos \theta_x \sin \theta_z & \sin \theta_x \sin \theta_z + \cos \theta_x \cos \theta_z \sin \theta_y & \sin \theta_y \cos \theta_z \\
\sin \theta_x \sin \theta_y + \cos \theta_x \cos \theta_z \sin \theta_y & \cos \theta_x \sin \theta_y \cos \theta_z & \cos \theta_x \cos \theta_y \cos \theta_z
\end{bmatrix}
\]

(3.5)

Figure 3.1: Transformation of the global translating, non-rotating axes to the debris principal axes using the rotation order of (a) pitch, (b) yaw, and (c) roll.
Deterministic Model Methodology

The 6-DoF deterministic debris trajectory model utilized for this study requires that the initial physical conditions of the plate (e.g., $X_0$, $V_0$, etc.) serve as the initial conditions of the beginning time step. Once the initial conditions are set, the relative velocity components of the debris with respect to the global translating, non-rotating axes are calculated:

$$U_G = V - W$$  \hspace{1cm} (3.6)

where $V$ is the debris velocity vector, [$V_X$, $V_Y$, $V_Z$], and $W$ is the uniform, horizontal and lateral wind vector, [$W_X$, 0, $W_Z$]. In order to define the flow angles of the debris (i.e., angle of attack and tilt angle) the relative debris velocity with respect to the global translating, non-rotating axes must be transformed into the relative velocity components along the debris principal axes, ($U_P$), by means of the global transformation matrix from Equation 5:

$$U_P = \begin{bmatrix} U_{XP} \\ U_{YP} \\ U_{ZP} \end{bmatrix} = T(\theta_X, \theta_Y, \theta_Z) \begin{bmatrix} U_{YG} \\ U_{YG} \\ U_{ZG} \end{bmatrix}$$ \hspace{1cm} (3.7)

The angle of attack ($\epsilon$) and the tilt angle ($\gamma$) are the observed angles between the relative debris velocity vector, and the debris principal $Y_PZ_P$-plane and the debris principal $X_PZ_P$-plane, respectively, as previously illustrated in Figure 2.7.

Results from wind tunnel tests performed by Richards et al. (2008) illustrated that the force coefficients ($C_F$) and moment coefficients ($C_{EM}$) are a function of the angle of
attack, the tilt angle, and the plate geometry \((G)\), in this case the side length ratio, which is defined as the ratio of the two debris dimensions perpendicular to the principal debris axis of interest with the larger dimension as the numerator. Several current studies (Baker 2007; Holmes et al. 2006; Kordi et al. 2010; Lin 2005; Lin et al. 2007; Martinez-Vazquez et al. 2009; Scarabino and Giacopinelli 2010) agree with the classification and utilization of the force and moment coefficients as a function of the angle of attack and the plate geometry; however, the effect of the tilt angle on the force and moment coefficients, which is presumably caused by the attachment of vortex structures to the leading edges of the plate at certain flow angle combinations, has only recently been explored through wind tunnel testing (Richards et al. 2005; Richards et al. 2008; Richards 2010). A recent study by Noda and Nagao (2010) has provided experimental results to assert that the force and moment coefficients are a function of more than one flow angle based on the relationship between the relative debris velocity vector and the debris principal axes; however, their definition of the flow angles is fundamentally different from Richards et al. (2008) albeit appropriate within their respective 6-DoF debris trajectory models.

Due to the dependence of the force and moment coefficients on the calculation of the angle of attack and the tilt angle within the debris trajectory model, it was deemed appropriate that the randomness required for a probabilistic debris trajectory model would be incorporated into the deterministic model at these points. Equations (3.8) and (3.9) provide the basis for the transition of the deterministic debris trajectory model into the probabilistic debris trajectory model. Since the deterministic flight model algorithm is unable to account for the random debris fields typically observed after wind storms, the
flow angles are modeled as random variables to account for the stochastic nature of the debris trajectory; therefore, it is assumed that the deterministic debris trajectory model will provide the mean value of the flow angles (i.e., mean angle of attack ($\bar{\varepsilon}$) and mean tilt angle($\bar{\gamma}$)) in order to sample the flow angles from a continuous statistical distribution:

$$\bar{\varepsilon} = \sin^{-1} \left( \frac{U_{xP}}{|U_P|} \right)$$

$$\bar{\gamma} = \tan^{-1} \left( \frac{U_{yP}}{U_{zP}} \right)$$

The sampled flow angles are used to select the appropriate force and moment coefficients from the experimental database compiled by Richards et al. (2008). This provides the necessary information to calculate the force applied to the plate along the debris principal axes as follows:

$$\mathbf{F}_p = \mathbf{C}_F (\varepsilon, \gamma, G) \frac{1}{2} \rho_a |U_p|^2 A_r \Rightarrow \begin{bmatrix} F_{pX} \\ F_{pY} \\ F_{pZ} \end{bmatrix} = \frac{1}{2} \rho_a |U_p|^2 \begin{bmatrix} C_{FX} (\varepsilon, \gamma, G) l_x l_z \\ C_{FY} (\varepsilon, \gamma, G) l_x l_y \\ C_{FZ} (\varepsilon, \gamma, G) l_x l_y \end{bmatrix}$$

where $\mathbf{F}_p$ is the external force applied to the plate along the debris principal axes, $\mathbf{C}_F$ is the force coefficients as a function of the angle of attack ($\varepsilon$), the tilt angle ($\gamma$), and the plate geometry ($G$), $\rho_a$ is the air density, and $A_r$ is the reference area of the plate, which is usually a projected frontal area, but since the dimensions of the plate always lie along the debris principal axes, it is the area perpendicular to the principal debris axis of interest.
The moments about the debris principal axes are handled in a similar fashion with the following equations:

\[ \mathbf{M}_p = \mathbf{M}_E + \mathbf{M}_D \quad (3.11) \]

where \( \mathbf{M}_p \) is the applied moment vector about the debris principal axes, \( \mathbf{M}_E \) is the external applied moment vector defined as:

\[
\mathbf{M}_E = \mathbf{C}_{EM} \left( \varepsilon, \gamma, G \right) \frac{1}{2} \rho_a |\mathbf{U}_p|^2 V_r \Rightarrow \left[ \begin{array}{c} M_{EX} \\ M_{EY} \\ M_{EZ} \end{array} \right] = \frac{1}{2} \rho_a |\mathbf{U}_p|^2 \left[ \begin{array}{c} C_{EMX} \left( \varepsilon, \gamma, G \right) \left( l_z A_y + l_y A_z \right) \\ C_{EMY} \left( \varepsilon, \gamma, G \right) \left( l_z A_y + l_y A_z \right) \\ C_{EMZ} \left( \varepsilon, \gamma, G \right) \left( l_z A_y + l_y A_z \right) \end{array} \right] \quad (3.12)
\]

where, \( \mathbf{C}_{EM} \) is the external applied moment coefficients as a function of the angle of attack (\( \varepsilon \)), the tilt angle (\( \gamma \)), and the geometry (\( G \)) of the plate, which is defined as the ratio of the two debris dimensions perpendicular to the principal debris axis of interest with the larger dimension as the numerator, \( \rho_a \) is the air density, |\( \mathbf{U}_p \)| is the magnitude of the relative velocity vector, and \( V_r \) is the debris reference volume (Noda and Nagao 2010). In addition to the external moments applied to the plate, Richards et al. (2008) states that without some form of damping, the plates would continue to rotate without bound; therefore, a damping moment vector, \( \mathbf{M}_D \), has been included in the external applied moments:

\[
\mathbf{M}_D = \mathbf{C}_{DM} \frac{1}{2} \rho_a \left( |\mathbf{U}_p| + \frac{1}{2} \mathbf{\omega} \| \mathbf{\ell}^2 \| \right) V_r \mathbf{\omega} \sqrt{\| \mathbf{\ell}^2 \|} \Rightarrow
\]
\[
\begin{align*}
\begin{bmatrix} M_{DX} \\ M_{DY} \\ M_{DZ} \end{bmatrix} &= \frac{1}{2} \rho_a \left( \begin{bmatrix} \mathbf{U}_P \end{bmatrix} + \left( \frac{1}{2} \omega_X \sqrt{l_Y^2 + l_Z^2} \right) \begin{bmatrix} C_{DMX} \omega_X \sqrt{l_Y^2 + l_Z^2} \end{bmatrix} \right) \left( l_Y A_Z + l_Z A_Y \right) \\
&+ \left( \frac{1}{2} \omega_Y \sqrt{l_X^2 + l_Z^2} \right) \begin{bmatrix} C_{DMY} \omega_Y \sqrt{l_X^2 + l_Z^2} \end{bmatrix} \left( l_X A_Z + l_Z A_X \right) \\
&+ \left( \frac{1}{2} \omega_Z \sqrt{l_X^2 + l_Y^2} \right) \begin{bmatrix} C_{DMZ} \omega_Z \sqrt{l_X^2 + l_Y^2} \end{bmatrix} \left( l_X A_Y + l_Y A_X \right)
\end{align*}
\]

where \( C_{DM} \) is the damping moment coefficient vector defined by Richards et al. (2008) as:

\[
C_{DM} = \begin{bmatrix} C_{DMX} \\ C_{DMY} \\ C_{DMZ} \end{bmatrix} = \begin{bmatrix} -0.05 \\ -0.185 \\ -0.185 \end{bmatrix},
\]

\( \ell \) is the debris reference length, and \( \mathbf{\omega} \) is the angular velocity vector:

\[
\mathbf{\omega} = \begin{bmatrix} \dot{\theta}_X \\ \dot{\theta}_Y \\ \dot{\theta}_Z \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_X + \dot{\theta}_Y \sin(\theta_Z) \\ \dot{\theta}_Z \sin(\theta_X) + \ddot{\theta}_Y \cos(\theta_X) \cos(\theta_Z) \\ \dot{\theta}_Z \cos(\theta_X) - \ddot{\theta}_Y \sin(\theta_X) \cos(\theta_Z) \end{bmatrix}
\]

where \( \theta_X, \theta_Y, \) and \( \theta_Z \) are the angular velocity components about the \( X_P, Y_P, \) and \( Z_P \) debris principal axes, respectively.

The database of force and moment coefficients utilized within Equations (3.10) and (3.12) are based on discrete values obtained from wind tunnel tests performed at the University of Auckland (Richards et al. 2008), in which the angle of attack and the tilt angle were incremented in specific intervals between 0 and 90 degrees during testing to develop a force and moment coefficient database. Due to the discrete nature of the
experimental database, a linear interpolation is performed between the discrete values in order to determine the force and moment coefficients for values of the angle of attack and the tilt angle other than those collected during testing.

The forces along the debris principal axes must be transformed into the forces along the global translating, non-rotating axes using the inverse of the global transformation matrix from Equation (3.5):

\[
\mathbf{F}_G = \begin{bmatrix}
F_{GX} \\
F_{GY} \\
F_{GZ}
\end{bmatrix} = \mathbf{T}(\theta_x, \theta_y, \theta_z)^{-1} \begin{bmatrix}
F_{PX} \\
F_{PY} \\
F_{PZ}
\end{bmatrix}
\]

This is necessary to determine the acceleration, and by extension the velocity, of the debris as follows:

\[
\ddot{\mathbf{X}} = \frac{1}{m} \mathbf{F}_G - g\mathbf{j}
\]

where \(m\) is the mass of the debris, \(g\) is the acceleration due to gravity, and \(\mathbf{j}\) is the unit vector along the global translating, non-rotating \(Y_G\)-axis, [0, 1, 0].

It is not necessary to transform the moments along the debris principal axes to the global translating, non-rotating axes since the angular accelerations and velocities, and the rate of change of the Tait-Bryan angles can be calculated along the debris principal axes using Euler’s equation for rigid body dynamics. Not transforming the moments about the debris principal axes to the global translating, non-rotating axes essentially reduces the number of calculations required as the mass moment of inertia vector (\(I\)) is constant along the debris principal axes:
\[ \dot{L}_p = M_d - \omega \times L_p \]  
(3.18)

where \( L_p \) is the angular momentum of the debris, which is defined as:

\[
L_p = I \omega = \begin{bmatrix} L_{px} \\ L_{py} \\ L_{pz} \end{bmatrix} = \begin{bmatrix} I_{xx} \omega_x \\ I_{yy} \omega_y \\ I_{zz} \omega_z \end{bmatrix}
\]  
(3.19)

\[
I = \begin{bmatrix} I_{xx} \\ I_{yy} \\ I_{zz} \end{bmatrix} = \begin{bmatrix} m(l_x^2 + l_z^2)/12 \\ m(l_x^2 + l_z^2)/12 \\ m(l_x^2 + l_y^2)/12 \end{bmatrix}
\]  
(3.20)

and \( M_d \) and \( \omega \) are the same as previously defined in Equations (3.12) and (3.15), respectively. Due to the coupling that is present in the basic equations of motion based on Newton’s second law of motion, Equations (3.15) and (3.17) through (3.19) are solved numerically using the Modified Euler’s Method (Appendix A). These solutions are then utilized to solve for the solutions of the next time step, and the process is repeated until the debris impacts the ground or another object (e.g., structure, vehicle, tree, etc.).

**Time Step Considerations**

The use of a numerical method such as the Modified Euler’s Method requires small time step increments to ensure that the approximation provided by the method is a reasonable assessment of the actual solution to the problem. However, a probabilistic debris trajectory model that is utilized within a larger simulation framework to assess building envelope failures must be simulated thousands of times to ensure that the simulation is a reasonable assessment of the physical situation. There must be a balance
between computational intensity and mathematical precision; therefore, a time step sensitivity study was performed on the deterministic debris trajectory model to determine an appropriate time step that would provide a reasonable amount of precision, since at this point the mathematical accuracy of the solution is limited by the deterministic debris trajectory model, yet be efficient in a large scale debris simulation. Figure 3.2 illustrates the results of the time step sensitivity study, in which six parameters (i.e., debris flight time, longitudinal position, longitudinal velocity, lateral position, lateral velocity, and vertical velocity at impact) were tested to determine the error induced into the system with an increase in time step above the initial value of 0.002s. Of the six parameters tested, the lateral position and velocity exhibit the most instability, especially for the case when the wind direction was 0°. This was caused by the values for the lateral position and velocity approaching zero in these cases on the order of \(10^{-14}\) in some instances, which caused large increases in the error based on equally small changes in the results.
Figure 3.3 illustrates the total error obtained from the individual parameter errors in Figure 3.2, and establishes the decision to choose 0.03s as the time step length for the probabilistic debris trajectory model based on the total error at 0.03s being relatively close to 10 percent (the total error at 0.03s would have been less than 10 percent if the unstable lateral position and velocity errors were not taken into account).

Figure 3.2: The sensitivity of the deterministic debris trajectory model to the duration of the time step. The initial time step duration was 0.002 s. All values taken at ground impact.
Figure 3.3: The total error and the average total error for all wind directions based on an increasing time step duration within the deterministic debris trajectory model. The initial time step duration was 0.002 s. All values taken at ground impact.
CHAPTER FOUR

PROBABILISTIC DEBRIS TRAJECTORY MODEL

Transition from Deterministic to Probabilistic

Limited experimental data on the statistical distribution of the debris flow angles during flight required that certain assumptions be made at the outset of the transition to a probabilistic debris trajectory model. It was assumed that the flow angles had an equal chance of falling on either side of the mean as calculated by the deterministic debris trajectory model; therefore this warranted the assumption that the appropriate statistical distribution was a normal distribution for both flow angles; therefore:

\[ \epsilon = N(\bar{\epsilon}, \sigma_{\epsilon}) = N(\bar{\epsilon}, \text{COV}_{\epsilon}) \]  \hspace{1cm} (4.1) \\
\[ \gamma = N(\bar{\gamma}, \sigma_{\gamma}) = N(\bar{\gamma}, \text{COV}_{\gamma}) \]  \hspace{1cm} (4.2)

where \( N(.) \) represents the normal distribution, \( \bar{\epsilon} \) and \( \bar{\gamma} \) are the mean angle of attack and tilt angle, respectively (Equations (3.8) and (3.9)). \( \sigma \) is the standard deviation, and \( \text{COV} \) is the coefficient of variation of the flow angles. The \( \text{COVs} \) are introduced to characterize the inherent randomness of debris flight due to wind turbulence, gustiness, etc. Due to the application of the Modified Euler’s method depending on the beginning of time step values and an end of time step estimate, the random flow angle values used to select the experimental force and moment coefficients were sampled at the beginning and end of each time step.
Coarse Parametric Study

In order to determine appropriate coefficient of variation values to be used to calculate the standard deviation that would be used in conjunction with the mean values of the flow angles (Equations (3.8) and (3.9)) to sample from a normal distribution, a coarse parametric study was performed by varying the values of the COV for the angle of attack and the tilt angle in increasing increments of 0.1 from a minimum COV equal to 0.1 up to a maximum COV equal to 2 for five wind directions: 0°, 15°, 30°, 45° and 60°. The values for the wind directions and the debris initial conditions input into the debris trajectory model (see Table 4.1) were deliberately chosen to permit validation of the probabilistic model through a direct comparison to test data from research on the trajectories of sheathing panels under high winds (Visscher and Kopp 2007; Kordi et al. 2010). The change in the wind direction as described by Visscher and Kopp (2007) and Kordi et al. (2010) was equivalent to changing the initial yaw \( \theta_{\text{yaw}} \) of the roof sheathing panel (Figure 3.1) within the probabilistic debris trajectory model. The initial position values provided in Table 4.1 are in reference to the geometric centroid of the roof sheathing panel. Since the scaled house model in Kordi et al. (2010) was rotated to represent a change in wind direction within the wind tunnel, the assumption was made in the probabilistic debris trajectory model that the geometric centroid of the sheathing panel remained in the same location with each subsequent change in the wind direction.

The scaled house model roof pitch was 4:12, which provided an angle from the horizontal of 18.4°, but based on the definition of the pitch angle \( \theta_{\text{z}} \) in Figure 3.1, the initial pitch of the roof sheathing plate implemented in the probabilistic debris trajectory
model was actually $90^\circ - 18.4^\circ = 71.6^\circ$. The uniform horizontal wind velocities ($W_X$) reported in Table 4.1 represented the 3-sec gust failure wind speed ($\hat{U}_H$) at the mean roof height observed by Kordi et al. (2010), and were reported to fit a Gumbel distribution (assumed to be a Type I Smallest extreme value distribution), which was taken into account within the probabilistic debris trajectory model. Debris flight times are generally on the order of a few seconds once flight has been initiated (Holmes 2004; Lin et al. 2007); therefore, it is reasonable to assume that the wind speed at the initiation of flight, in this case the release of the roof sheathing panel into the wind field after failure of the fasteners, remained constant for the duration of the debris flight (Lin 2005).

Table 4.1: Initial conditions of a 1.2 m x 2.4 m x 12.7 mm roof sheathing panel.

<table>
<thead>
<tr>
<th>Wind Speed ($W_X$)</th>
<th>Initial Debris Location</th>
<th>Initial Debris Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (m/s)</td>
<td>Std Dev (m/s)</td>
<td>$\mu$</td>
</tr>
<tr>
<td>56</td>
<td>2.2</td>
<td>55.0</td>
</tr>
<tr>
<td>45</td>
<td>1.4</td>
<td>44.4</td>
</tr>
<tr>
<td>41</td>
<td>1.6</td>
<td>40.3</td>
</tr>
<tr>
<td>40</td>
<td>1.5</td>
<td>39.3</td>
</tr>
<tr>
<td>38</td>
<td>1.4$^a$</td>
<td>37.3</td>
</tr>
</tbody>
</table>

$^a$ Designates assumed value not obtained from Kordi et al. (2010).

The ultimate goal of the coarse parametric study was to identify the combinations of the flow angle COVs that minimized the square root of the sum of the squares (SRSS) error between the simulated and full-scale experimental debris landing location statistics (Kordi et al 2010; Visscher and Kopp 2007):
\[ \varepsilon_x = \frac{\bar{x}_{\text{simulation}} - \bar{x}_{\text{data}}}{\bar{x}_{\text{data}}} = \frac{\bar{x}_{\text{simulation}}}{\bar{x}_{\text{data}}} - 1 \] (4.3)

\[ \varepsilon_{\text{total}} = \sqrt{\varepsilon_{\mu}^2 + \varepsilon_{\sigma}^2 + \varepsilon_{\lambda}^2} \] (4.4)

where \( \varepsilon_x \) is the error for a specific parameter represented by the variable \( x \), and \( \varepsilon_{\text{total}} \) is the SRSS error between the mean \( (\mu) \), standard deviation \( (\sigma) \) and skewness \( (\lambda) \) of the simulated data and the test data, respectively. The mean, standard deviation and the coefficient of skewness were calculated as follows:

\[ \mu = \frac{1}{n} \sum_{i=1}^{n} x_i \] (4.5)

\[ \sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu)^2} \] (4.6)

\[ \lambda = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^3 \left( \frac{1}{\sqrt{\sum_{i=1}^{n} (x_i - \mu)^2}} \right)^3 \] (4.7)

where \( n \) is the number of data points, \( i \) is an index, \( \mu \) is the mean, \( \sigma \) is the standard deviation, and \( \lambda \) is the coefficient of skewness of the data. It should be noted that for the coarse parametric study, Equation (4.4) is reduced to calculate the total error for only the first two terms in the equation, namely, the mean and standard deviation of the landing location in the longitudinal direction (along the \( X_e \)-axis), and in the lateral direction (along the \( Z_e \)-axis).
The upper limit COV value was chosen as 2 due to the phenomenon witnessed from a preliminary parametric sweep to determine the individual effect of the angle of attack and the tilt angle upon the total error ($\varepsilon_{total}$) as seen in Figure 4.1. It is evident, especially for the cases when the initial yaw (i.e., wind direction) is equal to $0^\circ$ and $15^\circ$, that as the COV utilized to sample the flow angles approaches 2 the total error increases significantly, and therefore, within this study it was desirable to not only select COV values for the angle of attack and the tilt angle that minimized the total error, but that also minimized the COV values of the angle of attack and the tilt angle themselves.

Figure 4.1: The effect of (a) the COV of the tilt angle on the total error with the COV of the angle of attack $= 0$, and (b) the COV of the angle of attack with the COV of the tilt angle $= 0$ at each tested wind direction.
Figure 4.2 illustrates several of the approximating surfaces from the coarse parametric study that were plotted to visually select specific ranges of the COV values of the angle of attack and the tilt angle for inclusion in the fine parametric study. The darkest areas of the surfaces provide the minimum values of the total error. Further evidence of the instability of the total error as the COV of the flow angles increases is apparent in Figure 4.2a, in which the wind direction is 0° (i.e., perpendicular to the initial position of the sheathing). The decrease in the instability of the total error with an increase in the wind direction (i.e., initial yaw) could be attributed to the reduction in the normal force on the plate at the higher wind directions.
Figure 4.2: Surface plots of the total error used in the coarse parametric study for wind directions of (a) 0°, (b) 15°, (c) 30°, and (d) 45°.
Number of Simulations Required

To determine the number of simulations required to provide sufficient data to determine the COV for the flow angles, the COV of the longitudinal landing impact position for a wind direction equivalent to 0° was calculated and plotted against the number of simulations. Simulations were performed until the COV of the longitudinal landing impact position for a 0° wind direction stabilized to the calculated mean of the COV to at least within a 3 percent error of the mean. It can be seen from Figure 4.3 that the COV of the longitudinal landing impact location reaches equilibrium with the mean value of the COV within 3 percent error at a minimum of 1500 simulations. It was assumed that the number of simulations required for the COV of the longitudinal landing impact location to reach equilibrium extended to the remaining wind directions within the debris trajectory model.

Fine Parametric Study

The fine parametric study consisted of establishing a range of the COV for the flow angles for each wind direction tested within the simulation (see Table 4.2). These ranges of the COV were determined from the surface plots that were created in the coarse parametric study; however, for the fine parametric study the number of simulations was increased from 100 to 1500 simulations as determined in the number of simulations study. The fine parametric study also included the skewness of the data (Equation (4.7)) in the calculation of the total error, in addition to the mean and standard deviation from the coarse parametric study. It should be noted that it was not necessarily the combination
of the COV of the flow angles that provided the overall lowest total error that were selected as the final values. A combination of the minimum COV of the flow angles, a visual inspection of the landing location plots, and a visual verification of the debris trajectory paths (Figure 4.5) were taken into consideration to ensure that the debris trajectories simulated by the probabilistic debris trajectory model agree with the trajectories reported in physical and numerical models (Richards et al 2008; Kordi et al. 2010).

Figure 4.3: Required number of simulations based on the COV of the longitudinal landing impact position.
Table 4.2: COV range identified by the surface plots in the coarse parametric study.

<table>
<thead>
<tr>
<th>Wind Direction (°)</th>
<th>Angle of Attack COV</th>
<th>Tilt Angle COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3 – 0.8</td>
<td>0.1 – 0.3</td>
</tr>
<tr>
<td>15</td>
<td>0.3 – 0.5</td>
<td>0.4 – 0.6</td>
</tr>
<tr>
<td>30</td>
<td>0.2 – 0.4</td>
<td>1.0 – 1.5</td>
</tr>
<tr>
<td>45</td>
<td>0 – 0.6</td>
<td>0.7 – 2.0</td>
</tr>
<tr>
<td>60</td>
<td>0.7 – 0.9</td>
<td>1.0 – 1.2</td>
</tr>
</tbody>
</table>

Figure 4.4 displays the final results from the fine parametric study. To provide the full 180° spectrum of COV values required within the probabilistic debris trajectory model, the assumption was made that the model would perform the same for the corresponding negative values of the wind direction (i.e. clockwise rotation about the positive Y-axis from 0° to -90°). Tests results from Kordi et al. (2010) at 75° and 90° determined that none of the panels took flight after failure, but rather landed back on the roof of the house model; therefore, since ground impact locations were unavailable for analysis, the COV for wind directions greater than 60° and less than -60° were assumed to be constant up to and including +/-90°. For wind directions in between those shown in Figure 4.4, linear interpolation was used within the probabilistic debris trajectory model to calculate the values of the COV of the angle of attack and the tilt angle.
Figure 4.4: The final representation of the COV of the angle of attack and the tilt angle utilized within the probabilistic debris trajectory model as determined from the fine parametric study.

Figure 4.5: Random debris paths of a 1.2m x 2.4m x 12.7mm roof sheathing panel with identical initial conditions.
CHAPTER FIVE

PROBABILISTIC MODEL VERIFICATION

Generally, most wind-borne debris research considers only subsystems of a building individually (e.g., roof sheathing, shingles, etc.) rather than testing the entire complex system as a whole (Surry et al. 2005, Kordi et al. 2010). This basic approach is insufficient in that it does not address the effect of the building aerodynamics, and the local velocities on the roof and in the wake on the flight of wind-borne debris (Kordi et al. 2010). Therefore, a ‘failure model’ has been employed by several recent research studies to take into account the influence imposed on wind-borne debris flight by the aerodynamics of the building and the local roof and building wake velocities (Surry et al. 2005, Visscher and Kopp 2007, Kareem 2008, and Kordi et al. 2010). Typically, numerical models that utilize a uniform, horizontal wind flow are unable to account for the variations in the velocity field due to the aerodynamics of the building and the local velocities. The probabilistic wind-borne debris model developed in this study accounts for the aleatoric uncertainty of the wind velocity field by sampling from a normal distribution with an appropriate COV of the flow angles that produce results comparable to experimental test data produced by the scaled debris flight tests of Visscher and Kopp (2007) and Kordi et al. (2010). The aforementioned probabilistic wind-borne debris model has been developed into a computer program which is capable of generating random debris flight paths.
Model Debris Position Verification

The values of the COV of the angle of attack and the tilt angle were verified by plotting the simulated ground impact locations with the test data provided by Kordi et al. (2010) as seen in Figure 5.1. The modes of flight identified by Visscher and Kopp (2007) and Kordi et al. (2010) (e.g., “translational”, “auto-rotational”, “3D spinning”, and “falling down”) were not tracked in this study; however, it can be seen in Figure 5.1 that regardless of the mode of flight illustrated during the simulated flight of the roof sheathing panels, there is a good agreement between the simulated ground impact locations and that tested by Kordi et al. (2010).

Table 5.1 provides a comparison of the impact location data statistics of the test data (Kordi et al. 2010) and the probabilistic debris trajectory model. As seen in Table 5.1, the simulated data is slightly overestimated or underestimated for each of the four data statistics (i.e., the mean and standard deviation of the longitudinal and lateral data respectively). As previously stated, the COV of the flow angles were selected to minimize this over/underestimation of the debris landing locations, and generally agree well with the wind tunnel data (Kordi et al. 2010).

Figure 5.2 shows the longitudinal debris impact locations as a function of the wind failure velocity (flight initiation wind speed). This plot reiterates the findings in Kordi et al. (2010) that the variation in the mean wind velocity are not a dominant factor affecting the distribution of the flight distances and the debris scatter.
Figure 5.1: Comparison of simulated ground impact locations of a 1.2 m x 2.4 m x 12.7 mm roof sheathing panel to test data (Kordi et al. 2010) for wind directions of (a) 0°, (b) 15°, (c) 30°, (d) 45°, and (e) 60°.

Figure 5.2: Longitudinal debris impact location as a function of wind failure velocity for wind directions of (a) 0°, (b) 15°, (c) 30°, (d) 45°, and (e) 60°.
Figure 5.3 depicts a vertical slice of the sheathing panel flights at longitudinal locations of 20 m and 30 m for wind directions of $0^\circ$, $15^\circ$, $30^\circ$, and $45^\circ$. The plot of $60^\circ$ is not shown since none of the roof sheathing panels attained a longitudinal position greater than or equal to 30 m. Generally, these plots agree well for the lateral positioning of the data; however, the probabilistic debris trajectory model exhibits an approximately 2 m increase in the maximum vertical range over the physical test data plotted by Kordi et al. (2010).

<table>
<thead>
<tr>
<th>Wind Direction</th>
<th>$X_{\text{avg}}$</th>
<th>$Z_{\text{avg}}$</th>
<th>$X_{\sigma}$</th>
<th>$Z_{\sigma}$</th>
<th>$X_{\text{avg}}$</th>
<th>$Z_{\text{avg}}$</th>
<th>$X_{\sigma}$</th>
<th>$Z_{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>33</td>
<td>5</td>
<td>19</td>
<td>5</td>
<td>36</td>
<td>3</td>
<td>17</td>
<td>7</td>
</tr>
<tr>
<td>$15^\circ$</td>
<td>36</td>
<td>2</td>
<td>12</td>
<td>5</td>
<td>34</td>
<td>1</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>43</td>
<td>9</td>
<td>9</td>
<td>3</td>
<td>39</td>
<td>10</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>31</td>
<td>11</td>
<td>15</td>
<td>4</td>
<td>27</td>
<td>13</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>13</td>
<td>10</td>
<td>6</td>
<td>4</td>
<td>12</td>
<td>9</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 5.1: Comparison of simulated impact location statistics to wind tunnel data.
It was imperative that the debris flight trajectories were plotted and analyzed visually to ensure that the probabilistic model was providing reasonable results. Unfortunately, there are few photographic examples of actual debris trajectories with corresponding flight data available; as a result, comparisons were made to scale model photographs of roof sheathing panel failures from Visscher and Kopp (2007), and the calculated deterministic debris trajectories from Richards et al. (2008). Figure 5.4 illustrates a simulated flight trajectory of a 1.2 m by 2.4 m by 12.7 mm thick roof sheathing panel.
The setup within the probabilistic debris trajectory model is the same as presented in Table 4.1 for a wind direction of 0°. The result of Figure 5.4 compares well with the results of Figure 5.1a, as the majority of the ground impact locations are contained within an approximately 15 m to 35 m range in the longitudinal direction.

Figure 5.4: Simulated flight of a 1.2 m x 2.4 m x 12.7 mm (side length ratio = 2) roof sheathing panel in a 56 m/s uniform horizontal wind, and wind direction = 0°.
Model Debris Velocity Verification

Debris velocity at impact is important in determining either the momentum or available energy of the debris at the time of impact. Typically, in many previous 2D debris flight trajectory models, the horizontal velocity is the parameter of interest in the determination of the impact speeds; however, as epistemic uncertainty has decreased with the advent of actual 6-DoF trajectory models, debris impact research is capable of looking at the magnitude of the resultant debris translational velocity ($u_{mag}$) as defined by Kordi et al. (2010):

$$u_{mag} = (u^2 + v^2 + w^2)^{0.5}$$  \hspace{1cm} (5.1)

where $u =$ the longitudinal (horizontal) component, $v =$ the vertical component, and $w =$ the lateral component of the debris velocity.

In order to compare the values of the magnitude of the resultant debris translational velocity, it must be normalized by the wind speed experienced by each piece of debris during the flight. This normalization leads to a dimensionless velocity quantity (Equation 5.2) as first proposed by Tachikawa (1988) and is a method that is used extensively in current research (e.g., Baker 2007, Holmes 2004, Holmes et al. 2006, Karimpour and Kaye 2010, Lin 2005, Lin et al. 2006, and Lin et al. 2007):

$$\bar{u}_{mag} = \frac{u_{mag}}{\hat{U}_H}$$  \hspace{1cm} (5.2)

where $\hat{U}_H$ is the peak factor 3-sec gust failure wind speed at the mean roof height.
Table 5.2 illustrates a comparison of the experimental and simulated dimensionless magnitudes of the resultant velocities that occurred at ground impact. Generally, there is good agreement between the experimental and the simulated data. There is slightly less variability in the dimensionless resultant velocities; however, this has been attributed to applying the peak factor 3-sec gust wind speed reported by Kordi et al. (2010), rather than a measured wind speed at the location of the sheathing panel.

Table 5.2: Comparison of the experimental and simulated dimensionless magnitude of the resultant debris velocities

<table>
<thead>
<tr>
<th>Wind Direction</th>
<th>Kordi et al (2010)</th>
<th>Simulated Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \left( \bar{u}<em>{mag} \right)</em>{avg} )</td>
<td>( \left( \bar{u}<em>{mag} \right)</em>{\sigma} )</td>
</tr>
<tr>
<td>0°</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>15°</td>
<td>0.73</td>
<td>0.21</td>
</tr>
<tr>
<td>30°</td>
<td>0.58</td>
<td>0.24</td>
</tr>
<tr>
<td>45°</td>
<td>0.43</td>
<td>0.20</td>
</tr>
<tr>
<td>60°</td>
<td>a</td>
<td>a</td>
</tr>
</tbody>
</table>

*a Denotes values not reported by Kordi et al (2010).

Figure 5.5 illustrates the dimensionless velocity quantities as seen at a vertical slice of the sheathing panel flights at longitudinal locations of 20 and 30 m for wind directions of 0°, 15°, 30°, and 45°. As with Figure 5.3, the plot of 60° is not shown since none of the roof sheathing panels attained a longitudinal position greater than or equal to 30 m. The range of the simulated roof sheathing panel speeds typically agrees well with the physical scale model data reported by Kordi et al (2010) for the wind directions of 15° and 30°; however, there appears to be less variability in the simulated panel speeds, and a slightly
higher (~2m) vertical position at both 20 m and 30 m longitudinal positions similar to the results in Figure 5.3. The simulated data does exhibit the same upward trend for roof sheathing panel speeds and increase in roof sheathing panel speeds with an increase in longitudinal position from 20 m to 30 m for wind directions of 15° and 30° as reported by Kordi et al (2010). These variations in panel speeds and vertical location could be attributed to the vortices and building wake effects experienced by the physical test data as the roof sheathing panel left the house model. The probabilistic debris trajectory model only minimized the error associated with the debris landing locations and neglected any influence that the debris velocity could have imposed in selecting the COV of the flow angles; however, after comparing the simulated data to test data, the slight differences in speed between the simulated and the physical data may not be significant if enough simulations are performed in a typical impact study.
Kordi et al (2010) observed that the dimensionless magnitudes of the resultant debris velocities fit a lognormal distribution when measured at the scale model house eave height (debris headed down from point of maximum trajectory towards the ground), and at ground impact. For comparison, Figure 5.6 depicts the dimensionless magnitude of the resultant debris translational velocity at ground impact for wind directions of 0°, 15°, 30°, 45°, and 60° at (top) longitudinal position = 20 m and (bottom) longitudinal position = 30 m.

Figure 5.5: Dimensionless panel velocities as a function of vertical position for wind directions of (a) 0°, (b) 15°, (c) 30°, (d) 45°, and (e) 60° at (top) longitudinal position = 20 m and (bottom) longitudinal position = 30 m.

Kordi et al (2010) observed that the dimensionless magnitudes of the resultant debris velocities fit a lognormal distribution when measured at the scale model house eave height (debris headed down from point of maximum trajectory towards the ground), and at ground impact. For comparison, Figure 5.6 depicts the dimensionless magnitude of the resultant debris translational velocity at ground impact for wind directions of 0°, 15°, 30°, 45°, and 60° fitted with both a lognormal probability density function (PDF) and the
corresponding cumulative distribution function (CDF). As with Kordi et al (2010), the
dimensionless magnitude of the velocities at ground impact were fit with a lognormal
distribution in most cases. At a level of significance assumed to be 0.05, a Kolmogorov-
Smirnov one-sample test (KS) confirmed that all cases, except for a wind direction of 15°
(Figure 5.6b), were likely to come from a lognormal distribution. Upon closer inspection
of Figure 5.1b, there appears to be an area at approximately 19 m that has a cluster of
debris impacts that could explain the higher number of low dimensionless velocities at
impact in the PDF of Figure 5.6b. This could be attributed to some instability in the force
coefficients obtained at earlier intervals of the COV for a wind direction of 15° as
portrayed in Figure 4.1, which could be related to fluctuations of the experimental force
coefficients.
Since the KS test determined that the dimensionless velocity distribution at 15° was unlikely to be from a lognormal distribution, Figure 5.7 compares the fit of a Type I Largest and Smallest (Gumbel) distribution to the fit of the lognormal distribution. From the figure it would appear that the Type I Largest (Gumbel) distribution fits the data better than either the Type I Smallest or the lognormal distributions; however, the histogram of the magnitude of the dimensionless resultant velocity data appears to exhibit

Figure 5.6: Probability distribution function (PDF) and cumulative distribution function (CDF) of the magnitude of the dimensionless resultant translational velocity of a 1.2 m x 2.4 m x 12.7 mm roof sheathing panel at ground impact for wind directions of (a) 0°, (b) 15°, (c) 30°, (d) 45°, and (e) 60° simulated within the probabilistic debris trajectory model.
a slight bimodal tendency; therefore, a KS test was performed on each of the distributions to determine if any can be verified as a best fit within a level of significance equal to 0.05.

Table 5.3 provides the results from the KS test of the distributions in Figure 5.7. While none of the three distributions can be verified as a likely distribution at a level of significance equal to 0.05, the KS test has confirmed Figure 16 in that the Type I Largest

Figure 5.7: Determination of the best fit probability distribution function (PDF) and cumulative distribution function (CDF) of the magnitude of the dimensionless resultant translational velocity of a 1.2 m x 2.4 m x 12.7 mm roof sheathing panel at ground impact for a wind direction of 15°.

Table 5.3 provides the results from the KS test of the distributions in Figure 5.7. While none of the three distributions can be verified as a likely distribution at a level of significance equal to 0.05, the KS test has confirmed Figure 16 in that the Type I Largest
(Gumbel) distribution is a much better fit than the Type I Smallest or lognormal distributions.

Table 5.3: Kolmogorov-Smirnov (KS) one-sample test best fit results for the magnitude of the dimensionless resultant translational velocity of a 1.2 m x 2.4 m x 12.7 mm roof sheathing panel at ground impact for a wind direction of 15°

<table>
<thead>
<tr>
<th>Wind Direction (°)</th>
<th>KS critical</th>
<th>KSLognormal</th>
<th>KS\textsubscript{Gumbel Largest}</th>
<th>KS\textsubscript{Gumbel Smallest}</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.1340</td>
<td>0.2387</td>
<td>0.1384</td>
<td>0.2790</td>
</tr>
</tbody>
</table>

Level of Significance = 0.05
CHAPTER SIX

APPLICATIONS OF THE PROBABILISTIC MODEL

Several current research studies (e.g., Holmes et al. 2006, Lin et al. 2006, Lin et al. 2007, and Baker 2007) have sought to include the probabilistic aspects of debris impact into design procedures and test protocols (Lin et al. 2007). In order to accomplish this, advancements in numerical debris trajectory models and physical testing are needed to continuously improve upon our methods of debris impact risk assessment. However, there is a substantial lack of data for the debris impact of sheet-type debris. An inability to replicate debris impact speeds and orientations with sheet-type debris required standards and codes (e.g., ASTM E 1886-05, ASTM E 1996-09) to adopt a 4.1 kg, 50 mm x 100 mm dimensional lumber specimen as the basis for impact testing (Yazdani et al. 2006). Therefore, this Chapter is dedicated to identifying what information is available in the application of the proposed probabilistic debris trajectory model, and what will be needed from future research in order to calibrate, verify, and improve upon the current model.

Comparison to Current 2D Debris Impact Approximations

Holmes (2010) has stated that the horizontal debris velocities of wind-borne debris are the principal quantities of interest in calculating kinetic energy and momentum at impact; therefore, many of the current 2D debris trajectory models (e.g., Baker 2007 and Lin et al. 2007) have numerically and experimentally illustrated that the horizontal debris
velocity of wind-borne debris follows an exponential curve that is dependent on the horizontal distance travelled by the debris, and not the wind speed:

\[
\bar{u} \equiv 1 - \exp\left[ -bx \right]
\]  \hspace{1cm} (6.1)

where,

\[
b = \sqrt{\frac{\rho_a C_{D,av}}{m}} = \sqrt{\frac{\rho_a C_{D,av}}{\rho_m t}}
\]  \hspace{1cm} (6.2)

Notice in Equation (6.2) that the horizontal debris velocity is only dependent on the plate thickness as shown by Wills et al. (2002).

On the surface, it seems that a comparison between the proposed probabilistic debris trajectory model and the estimates from the Lin and Baker equation (Equation (6.1)) would be fairly straightforward, however, the data used to calibrate the model from Kordi et al. (2010) was for a scaled plate that assumed that there were shingles still attached to the roof sheathing panel. This is a reasonable assumption based on post-disaster damage surveys, however, this resulted in a significantly larger material density \( (\rho_m) \) for the roof sheathing panel used in the calibration of the proposed model as opposed to the dimensionally comparable plates utilized by Lin et al. (2006) in their best fit equation (Equation (6.1)). Examination of Equation (6.2) illustrates that the horizontal debris velocity is inversely proportional to the material density multiplied by the plate characteristic length (i.e., the thickness of the plate); therefore, one would expect the results of Figure 6.1, in which the dimensionless horizontal debris speeds provided by the probabilistic debris trajectory model are less than those approximated by Equation (6.1).
However, the probabilistic debris trajectory model does follow the same exponential trend as the dimensionless horizontal debris velocity approaches an asymptote at $\bar{u} = 1$ (i.e., horizontal debris speed is equal to the wind speed). It is very likely during an extreme wind event that wind-borne debris released will be a composite of roofing materials or some combination other than a clean sheet of sheathing; therefore, it is possible that the proposed debris trajectory model can be utilized to formulate a modification factor to account for the change in the horizontal debris speeds of these wind-borne “composites.”

Figure 6.1: Dimensionless horizontal debris speed at impact as a function of horizontal displacement.
Assessment of Wind-borne Debris Impact Methodology

It is generally assumed that the horizontal debris speed component is the main contributor to damage from debris impacts, however, it seems reasonable to the author that there exists a scenario during an extreme wind event in which the resultant speed of the debris trajectory could be the normal component to the impacted surface. Therefore, this study is a preliminary investigation to determine if the neglect of key parameters associated with impact mechanics (e.g., angular velocity, debris orientation, etc.) provides substantially different impact results from what is commonly estimated using simplified trajectory models, and whether further investigation is warranted based on these preliminary results. Figure 6.2 plots the mean (blue) results from Figure 6.1 in addition to the mean dimensionless resultant debris velocity (red) for each wind speed tested. An initial evaluation of Figure 6.2 seemed to be incorrect as the initial resultant debris velocities exceeded the wind speed at $U = 10 \text{ m/s}$. However, this is a graphical representation of the Tachikawa Number ($K$), which to reiterate is the ratio of the aerodynamic forces to the gravitational forces. At low wind speeds, the gravitational forces are dominant over the aerodynamic forces; therefore the vertical component of velocity at impact will be greater than the horizontal component, which depending on the height of the debris release can cause the resultant debris speed to exceed the wind speed. As the wind speed increases, the aerodynamic forces become dominate and the horizontal debris velocity becomes the dominant component of the debris velocity; therefore, the dimensionless resultant debris velocity approaches the same asymptotic value as the dimensionless horizontal velocity.
Generally, the kinetic energy or the momentum of the debris is calculated to quantify the damage potential of wind-borne debris. Both measurements work well in debris impact damage assessments in specific situations. Typically, momentum is used in situations where the deformation of a surface can be assumed elastic (e.g., glass), however, HAZUS-MH, which is a risk assessment software package created by FEMA (2007) that contains models for estimating losses due to earthquakes, floods, and hurricanes, utilizes the linear kinetic energy to determine the potential damage of wind-borne debris:

\[
KE = \frac{1}{2} mV^2
\]  

(6.3)

Figure 6.2: Mean values of the dimensionless horizontal and resultant speeds.
where, $m$ is the mass of the debris and $V$ is the linear speed of the debris. Due to the popularity of using the kinetic energy as the measure of debris impact damage, this preliminary study will focus on that parameter.

Since many impact standards only test non-rotating rod-type debris perpendicular to the surface, the calculation of the linear kinetic energy (LKE) is a reasonable assumption, but it is not likely that all impacts during an extreme wind event will occur in this fashion. The majority of debris (rods and plates) will exhibit rotation about one up to all three of its axes. Figure 6.3 depicts the effect of including the rotational kinetic energy in the calculation in addition to adding the resultant debris velocity to the linear kinetic energy to obtain the total kinetic energy (TKE) of the debris:

$$TKE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2} \left( mv^2 + I_{xx}\omega_x^2 + I_{yy}\omega_y^2 + I_{zz}\omega_z^2 \right)$$

(6.4)

where $I$ is the mass moment of inertia of the debris and $\omega$ is the angular velocity (Hibbeler 2007). It should be noted that the mass moment of inertia is constant in this case because it is taken about the debris principal axes, as is the angular velocity. Figure 6.4 depicts the theoretical maximum linear kinetic energy (i.e., the horizontal velocity of the debris is equal to the wind speed) compared to the TKE of the debris. It is shown that there is the potential for the TKE to exceed the worst case scenario for the LKE given the debris flies far enough and fast enough. Figure 6.3 and Figure 6.4 have illustrated that the increase in the TKE is fairly substantial when considering other debris parameters and not just the horizontal velocity; future research should consider looking into this further.
Figure 6.3: The effect of the resultant debris velocity on the total kinetic energy.

Figure 6.4: Total kinetic energy of plate relative to the wind speed.
Building Envelope Impact Risk Assessments

Light-frame wood construction represents a significant portion of residential housing in North America, Oceania, and Europe, and is becoming more prevalent in other areas of the world such as Japan, China, and India. However, many of these areas are at significant risk for extreme wind events (i.e., hurricanes, typhoons or tropical cyclones depending on location); therefore, it is crucial that steps be taken to ensure that economic and societal losses are reduced through mitigation activities identified as a result of building envelope impact risk assessments (Figure 6.5). The proposed wind-borne debris model provides an effective method for predicting the variation of debris trajectories in a 3D space, which is imperative when performing regional building envelope impact risk assessments in which a large amount of debris sources and targets must be considered in the simulation; in addition, the model provides guidance on debris impact protection, and serves as a tool in the analysis and verification of design impact loads experienced by light-frame wood construction during extreme wind events.

Figure 6.5: Application of the probabilistic model in an impact risk assessment.
CHAPTER SEVEN

CONCLUSIONS

Summary and Conclusions

A deterministic 6-DoF debris trajectory model has been modified to perform as a probabilistic model in this paper. By adapting the model to provide results that match physical test data, the aleatoric uncertainty has been addressed by incorporating error usually associated with uniform, horizontal flow debris trajectory models into the coefficient of variation values used to sample the angle of attack ($\varepsilon$) and the tilt angle ($\gamma$) of the roof sheathing panel from a normal distribution.

Overall comparisons to current ‘failure model’ research have matched well in terms of debris flight trajectories and velocities. The proposed probabilistic model provides an effective method for predicting debris trajectories in a 3D space (Figure 5.4), which is imperative when performing regional building envelope impact risk assessment in which a large amount of debris sources and targets must be considered in the simulation. Though a true 6-DoF can be slightly more computationally intensive to implement, the proposed probabilistic model presents the opportunity to obtain a greater knowledge of an inherently complex system. A preliminary study regarding the current wind-borne debris impact methodology has illustrated that there is a substantial increase in the total kinetic energy of the debris impact when all of the dynamic parameters of the debris trajectory, translational and rotational, are considered; therefore the proposed probabilistic wind-
borne debris trajectory model can be used to analyze building envelope impact failures of light-frame wood structures due to wind hazards, provide guidance on debris impact protection, and serve as a tool in the analysis and verification of design impact loads experienced by structures during extreme wind events.

Recommendations for Future Research

There is an extensive amount of work that will need to be completed in future research to continually improve upon probabilistic debris trajectory and impact models; therefore, the following recommendations, ranked in order of the greatest contribution with the least degree of difficulty to conduct, to lesser contributions with a greater degree of difficulty to conduct, are made for continuing research:

- Based on the preliminary study into the assessment of the wind-borne debris impact methodology, further investigation into how the rotational components of the debris (e.g., angular acceleration, angular velocity, etc.) influence the impact calculations (i.e., kinetic energy and momentum) would be informative.

- The preliminary investigation and previous studies from Clemson University (Sciaudone 1996; Pietras 2000) recognize that the debris impact orientation plays a significant role in the damage potential of wind-borne debris; therefore, further investigation into the angle of debris impact relative to a surface (i.e., the normal vector of the surface), and the shape of the impacting surface of the
debris is required if continued progress is to be made in debris impact mitigation.

- Further investigation into a modification factor for composite wind-borne debris (e.g., roof sheathing panels with attached shingles or tiles, fascia boards with soffit attached, etc.).

- More studies similar in scope to Karimpour and Kaye (2010) and Kordi and Kopp (2011) are required to fully understand how a non-uniform wind field and building local effects can be taken into account in probabilistic debris trajectory models for sheet-type and rod-type debris. Some of the current assumptions (e.g., constant wind field over the short duration of debris flight, turbulence does not have a significant effect, etc.) would be inadequate for the inclusion of these types of wind field models.

- Further investigation is needed into how the “release mechanism” of the debris affects the initial conditions of the debris trajectory. In other words, does the springing action of debris affect the initial position, velocity or acceleration of the debris significantly.

In closing, several of the aforementioned recommendations for future work can be investigated using the proposed probabilistic wind-borne debris trajectory model, several of the recommendations will be required to continue to improve upon the model and ensure that the model provides accurate results, however, all these recommendations are beneficial to improving our knowledge and understanding of the field of wind-borne
debris trajectories and impacts, and how we can further mitigate the socioeconomic losses that occur with extreme wind events.
APPENDICES
Appendix A

Modified Euler’s Method

In order to obtain a solution to the equations of motion, it is necessary to utilize numerical methods to approximate the translational and rotational positions and velocities of the debris from Equations (3.15) and (3.17) through (3.19). The Modified Euler’s Method was chosen for this study since it is a single-step, explicit, second-order numerical technique for solving first-order ordinary differential equations (ODE). The following presents an algorithm for the application of the Modified Euler’s Method followed by an example in the context of this study (Gilat and Subramaniam 2008).

Initial conditions are utilized to solve for the initial values at the beginning of the time step; therefore, \( x_{i+1} = x_i + h \), and the dependent variable at the beginning of the time step is \( f(x_i, y_i) \). Estimate the value of the dependent variable at the end of the time step using Euler’s Method, \( y_{i+1}^{EU} = y_i + f(x_i, y_i)h \). Calculate the dependent variable at the end of the time step using the estimate, \( f(x_{i+1}, y_{i+1}) \), where \( y_{i+1} = y_{i+1}^{EU} \), and then calculate the numerical solution at \( x = x_{i+1} \):

\[
y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{EU})}{2} h
\]

(A.1)

Repeat the process for the remaining time steps until the process is complete.
The following example is the implementation of the Modified Euler’s Method to calculate the horizontal debris velocity of the debris for one iteration:

1. Calculate the horizontal debris velocity, $\dot{X}_i$,

2. Calculate the acceleration at the start of the time step from the initial force calculations, $\ddot{X}_i$,

3. Estimate the velocity at the end of the time step, $\dot{X}_e = \dot{X}_i + \ddot{X}_i \Delta t$

4. Calculate the acceleration at the end of the time step from the subsequent force calculations, $\ddot{X}_e$

5. Calculate the velocity at the start of the next time step,
   \[
   \dot{X}_{i+1} = \dot{X}_i + \frac{\dot{X}_i + \dot{X}_e}{2} \Delta t
   \]

6. Repeat the process for the subsequent time step
Appendix B

MATLAB Code

B.1: Probabilistic Wind-borne Debris Trajectory Model Code

% 6-DoF Probabilistic Wind-borne Debris Trajectory Model
%
% Written by: J. Michael Grayson
% Graduate Research Assistant
% Glenn Department of Civil Engineering
% Clemson University
% Email: jmgrays@g.clemson.edu
%
% Secondary contact: Dr. WeiChiang Pang
% Assistant Professor
% Glenn Department of Civil Engineering
% Clemson University
% Email: wpang@clemson.edu
%
% Adapted from a deterministic debris trajectory model provided by:
% Dr. Peter J. Richards
% Department of Mechanical Engineering
% University of Auckland
%
% Last Modified: Nov-25-2011
% by: J. Michael Grayson
%
% Data
% Short_Fourier.mat = Short Fourier series coefficient data

% External Functions
% Fourier_Series.m = Assigns short fourier series coefficients based on
% based on debris parameters
%
% aeroCoeff.m = uses bi-directional short fourier series proposed by
% Richards 2010 to create an array of aerodynamic force
% and moment coefficients based on the value of the
% angle of attack (epsilon) and the tilt angle (gamma)

% Variables
% User Input:
% nSims = the number of simulations
% windSpeed = uniform horizontal wind speed (m/s)
% Lx, Ly, Lz = length along the respective principal axes (m)
% debrisIdentifier = 'Plate' or 'Rod' - case insensitive
% density = density of the debris material (kg/m^3)
% initialX = Initial longitudinal position of debris centroid (m)
% initialY = Initial vertical position of debris centroid (m)
% initialZ = Initial lateral position of debris centroid (m)
% initialRoll = thetaX defined in Figure 3.1 (deg)
% initialYaw = thetaY defined in Figure 3.1 (deg)
% initialPitch = thetaZ defined in Figure 3.1 (deg)
% rhoAir = density of air (kg/m^3)
% g = acceleration due to gravity (m/s^2)

% Debris parameters:
% shapeIndex = debris identifier based on side length ratio of debris
% Ax, Ay, Az = Areas perpendicular to the principal axes (m^2)
% mass = mass of the debris (kg)
% Ixx, Iyy, Izz = mass moment of inertia about the debris axes (kg-m^2)

% Simulation parameters:
% CDMX, CDMY, CDMZ = damping moment coefficients (dimensionless)
% stall = inclusion of stall hysteresis - see Richards et al. 2008
% camber = inclusion of apparent camber - see Richards et al. 2008
% max_camber = maximum apparent camber - see Richards et al. 2008
% maxSteps = maximum number of data points to prevent endless loop
% delta_t = time step increment (s)
% epsCOV_array, gamCOV_array = COV values for epsilon and gamma
% initialYaw_array = array for interpolating epsilon and gamma
% epsilon, gamma = vector of values for epsilon and gamma
% epsilon_array, gamma_array = grid of 'epsilon' and 'gamma' vectors
% Fourier_fx, .fy, .fz = short Fourier series - see Richards 2010
% Forcex, .y, .z = force coefficients based on debris parameters
% Forcemx, .my, .mz = moment coefficients based on debris parameters
% t = debris flight time (s)
% Note: appended '_begin' or '_end' signifies start or end of time step
% x, y, z = debris translational position (m)
% u, v, w = debris translational velocity (m/s)
% thetaX, .Y, .Z = Tait-Bryan angles (deg) - see Figure 3.1
% omegaX, .Y, .Z = angular velocity about the debris axes (rad/s)
% thetaXdot, .Ydot, .Zdot = rate of change of Tait-Bryan angles (rad/s)
% UXG, UYG, UZG = relative velocity to wind along ground axes (m/s)
% UX, UY, UZ = relative velocity to wind along debris axes (m/s)
% URP = resultant relative debris velocity to wind (m/s)
% UXP_over_URP = relative velocity ratio (dimensionless)
% epsilonBeginMu, .endMu = mean value of epsilon (dimensionless)
% gammaBeginMu, .endMu = mean values of gamma (dimensionless)
% dyn_press = dynamic pressure (N/m^2)
% CoP = center of pressure (m)
% aspectRatio = effective aspect ratio (dimensionless)
% CFX, CFY, CFZ = force coefficients based on epsilon and gamma
% CFX_hyst = force coefficient including stall hysteresis
% CMX, CMY, CMZ = moment force coefficients based on epsilon and gamma
% FXP, FYP, FZP = forces on debris along the debris principal axes (N)
% FXG, FYG, FZG = forces on debris along the ground axes (N)
% udot, vdot, wdot = debris translational acceleration (m/s^2)
% Mx, My, Mz = moments on debris about the debris principal axes (N-m)
% omegaXdot, .Ydot, .Zdot = angular accel about debris axes (rad/s^2)

% Output:
% x, y, z = debris translational position during trajectory (m)
% u, v, w = debris translational velocity during trajectory (m/s)
% roll, yaw, pitch = Tait-Bryan angles during trajectory (rad)
% p, q, r = debris angular velocity during trajectory (rad/s)
% pdot, qdot, rdot = debris angular accel during trajectory (rad/s^2)

%% Preparation for Simulation
close all; clear; clc;

%% Seed the Random Number Generator
% Seed from computer internal clock - required for cluster computing
RandStream.setDefaultStream(RandStream('mt19937ar','seed',sum(100.*... clock)));

%% User Input: Example plate from Kordi et al. 2010
% Modify the variables in this section to suit user-defined situation
nSims = 1; % The number of simulations desired
windSpeed = 50; % Uniform horizontal windspeed along the 'X' axis (m/s)

% Debris dimensions
% Conditions that must be met for plates and rods: Lz > Ly > Lx
% Conditions that must be met for rods: Lz >= 10*Ly
Lx = 0.0127; % Length along the debris principal 'X_P' axis (m)
Ly = 1.2; % Length along the debris principal 'Y_P' axis (m)
Lz = 2.4; % Length along the debris principal 'Z_P' axis (m)
debrisIdentifier = 'Plate'; % Choose 'Plate' or 'Rod'
density = 1829; % This example includes density of shingles (kg/m^3)

% Input for Debris Initial Location
initialX = 0; % Longitudinal position (m)
initialY = 10; % Vertical position (m)
initialZ = 0; % Lateral position (m)

% Initial debris angles (Tait-Bryan angles)
% Defined in Figure 3.1
initialRoll = 0; % (deg)
initialYaw = 0; % (deg) % Kordi et al. 2010 tested: 0,15,30,45,60 deg
initialPitch = 71.5; % (deg)

% Constants
rhoAir = 1.2; % (kg/m^3)
g = 9.81; % (m/s^2)

%% Calculated Debris Properties
% Calculate side length ratio to determine shape index
% Shape Index
% 1 = 1 to 1 Rod
% 2 = 2 to 1 Rod
% 3 = 3 to 1 Rod
% 4 = 1 to 1 Plate
% 5 = 2 to 1 Plate
% 6 = 4 to 1 Plate
% Ranges for shape index recommended by Dr. Peter J. Richards of the
% University of Auckland, New Zealand
if strcmpi(debrisIdentifier, 'rod') && Ly/Lx <= 1.5
    shapeIndex = 1;
elseif strcmpi(debrisIdentifier, 'rod') && Ly/Lx > 1.5 && Ly/Lx <= 2.5
    shapeIndex = 2;
elseif strcmpi(debrisIdentifier, 'rod') && Ly/Lx > 2.5 && Ly/Lx <= 4.0
    shapeIndex = 3;
elseif strcmpi(debrisIdentifier, 'plate') && Lz/Ly <= 1.5
    shapeIndex = 4;
elseif strcmpi(debrisIdentifier, 'plate') && Lz/Ly > 1.5 && Lz/Ly <= 3.0
    shapeIndex = 5;
elseif strcmpi(debrisIdentifier, 'plate') && Lz/Ly > 3.0 && Lz/Ly <= 6.0
    shapeIndex = 6;
else
    error('Debris dimensions do not meet necessary conditions');
end;

% Calculate debris areas
Ax = Ly .* Lz;  % Area perpendicular to the 'X_P' axis (m^2)
Ay = Lx .* Lz;  % Area perpendicular to the 'Y_P' axis (m^2)
Az = Lx .* Ly;  % Area perpendicular to the 'Z_P' axis (m^2)

% Calculate debris mass
mass = density.*Lx.*Ly.*Lz;  % (kg)

% Mass moment of inertia about the debris principal axes
Ixx = mass.*(Ly.^2+Lz.^2)./12;  % (kg-m^2)
Iyy = mass.*(Lx.^2+Lz.^2)./12;  % (kg-m^2)
Izz = mass.*(Lx.^2+Ly.^2)./12;  % (kg-m^2)

% Damping coefficients (plates only)
if shapeIndex > 3
    CDMX = -0.05;  % (dimensionless)
    CDMY = -0.185; % (dimensionless)
    CDMZ = -0.185; % (dimensionless)
else
    CDMX = 0;  % (dimensionless)
    CDMY = 0;  % (dimensionless)
    CDMZ = 0;  % (dimensionless)
end;

% Determine inclusion of "stall" and "camber" based on shape index
% Only plates are subject to stall and camber
% 1 - includes stall or camber; 0 - excludes stall or camber
% Based on Richards et al. 2008
if shapeIndex > 3
    stall = 1;
camber = 1;
else
    stall = 0;
camber = 0;
end;

% Set maximum apparent camber
max_camber = 0.4; % Based on Richards et al. 2008

% Number of maximum possible time steps
maxSteps = 1000;

% Set simulation time step
delta_t = 0.03; % (s)

%% Declare Array of COVs for the Flow Angles
if initialYaw > -60 && initialYaw < 60
    epsCOV_array = [0.9, 0.6, 0.2, 0.3, 0.6, 0.3, 0.2, 0.6, 0.9];
    gamCOV_array = [1.0, 2.0, 1.4, 0.4, 0.3, 0.4, 1.4, 2.0, 1.0];
    initialYaw_array = [60, 45, 30, 15, 0, -15, -30, -45, -60];
    % Interpolate to determine values of COV for epsilon and gamma
    eps_COV = interp1(initialYaw_array, epsCOV_array, initialYaw);
    gam_COV = interp1(initialYaw_array, gamCOV_array, initialYaw);
else
    eps_COV = 0.9;
    gam_COV = 1.0;
end;

%% Declare Arrays for Flow Angles and Force Coefficients
% Epsilon is the angle of attack
epsilon = degtorad(-90:5:90);
% Gamma is the tilt angle
gamma = degtorad(0:5:360);

% Compile Short Fourier Series from "Fourier_Series" Function
[Fourier_fx,Fourier_fy,Fourier_fz,Fourier_mx,Fourier_my,Fourier_mz]=...
    Fourier_Series(shapeIndex);

% Compile Aerodynamic coefficient arrays from "aeroCoeff" Function
[Forcex,Forcey,Forcez,Forcemx,Forcemy,Forcemz]=aeroCoeff(Fourier_fx,...
    Fourier_fy,Fourier_fz,Fourier_mx,Fourier_my,Fourier_mz,...
    gamma,epsilon);

% Assign Flow Angles to "meshgrid" to interpolate force coefficients
% Create arrays of gamma and epsilon
[gamma_array,epsilon_array] = meshgrid(gamma,epsilon);
% Preallocation of Variables
% Debris flight time
t = zeros([1,maxSteps]);
% Debris translational position variables
x_begin = zeros([1,maxSteps]);
y_begin = zeros([1,maxSteps]);
z_begin = zeros([1,maxSteps]);

% Debris translational velocity variables
u_begin = zeros([1,maxSteps]);
v_begin = zeros([1,maxSteps]);
w_begin = zeros([1,maxSteps]);
u_end = zeros([1,maxSteps]);
v_end = zeros([1,maxSteps]);
w_end = zeros([1,maxSteps]);

% Debris translational acceleration variables
udot_begin = zeros([1,maxSteps]);
vdot_begin = zeros([1,maxSteps]);
wdot_begin = zeros([1,maxSteps]);
udot_end = zeros([1,maxSteps]);
vdot_end = zeros([1,maxSteps]);
wdot_end = zeros([1,maxSteps]);

% Tait-Bryan angle variables
thetaX_begin = zeros([1,maxSteps]);
thetaY_begin = zeros([1,maxSteps]);
thetaZ_begin = zeros([1,maxSteps]);
thetaXdot_begin = zeros([1,maxSteps]);
thetaYdot_begin = zeros([1,maxSteps]);
thetaZdot_begin = zeros([1,maxSteps]);
thetaXdot_end = zeros([1,maxSteps]);
thetaYdot_end = zeros([1,maxSteps]);
thetaZdot_end = zeros([1,maxSteps]);

% Debris rotational velocity variables
omegaX_begin = zeros([1,maxSteps]);
omegaY_begin = zeros([1,maxSteps]);
omegaZ_begin = zeros([1,maxSteps]);

% Debris rotational acceleration variables
omegaXdot_begin = zeros([1,maxSteps]);
omegaYdot_begin = zeros([1,maxSteps]);
omegaZdot_begin = zeros([1,maxSteps]);
omegaXdot_end = zeros([1,maxSteps]);
omegaYdot_end = zeros([1,maxSteps]);
omegaZdot_end = zeros([1,maxSteps]);

% Flow angle variables
epsilon_begin = zeros([1,maxSteps]);
epsilon_end = zeros([1,maxSteps]);
gamma_begin = zeros([1,maxSteps]);
gamma_end = zeros([1,maxSteps]);
%% Simulations
for a = 1:nSims
    % Calculate Debris Trajectory Path
    for b = 1:maxSteps % MaxSteps to prevent perpetual loop

        if b == 1
            % Set initial time
            t(b) = 0;

            % Initial debris position
            x_begin(b) = initialX;
            y_begin(b) = initialY;
            z_begin(b) = initialZ;

            % Initial debris velocities
            u_begin(b) = 0;
            v_begin(b) = 0;
            w_begin(b) = 0;

            % Initial Tait-Bryan Angles
            thetaX_begin(b) = degtorad(initialRoll);
            thetaY_begin(b) = degtorad(initialYaw);
            thetaZ_begin(b) = degtorad(initialPitch);

            % Initial rate of change of Tait-Bryan angles
            thetaXdot_begin(b) = 0;
            thetaYdot_begin(b) = 0;
            thetaZdot_begin(b) = 0;

            % Initial angular velocities at start of time step
            omegaX_begin(b) = thetaXdot_begin(b) + ...
                              thetaYdot_begin(b).*sin(thetaZ_begin(b));
            omegaY_begin(b) = thetaZdot_begin(b).*...
                              sin(thetaX_begin(b))+thetaYdot_begin(b).*...
                              cos(thetaZ_begin(b)).*cos(thetaX_begin(b));
            omegaZ_begin(b) = thetaZdot_begin(b).*...
                              cos(thetaX_begin(b))-thetaYdot_begin(b).*...
                              cos(thetaZ_begin(b)).*sin(thetaX_begin(b));

        else
            % Calculate cumulative debris flight time
            t(b) = delta_t.*(b-1);

            % Modified Euler Method
            % Debris position at start of time step
            x_begin(b) = x_begin(b-1) + (u_begin(b-1) +...
                                        u_end(b-1)).*0.5.*delta_t;
            y_begin(b) = y_begin(b-1) + (v_begin(b-1) +...
                                        v_end(b-1)).*0.5.*delta_t;
            z_begin(b) = z_begin(b-1) + (w_begin(b-1) +...
                                        w_end(b-1)).*0.5.*delta_t;

        end

    end
end
% Debris velocities at start of time step
u_begin(b) = u_begin(b-1) + (udot_begin(b-1) + udot_end(b-1)).*0.5.*delta_t;
v_begin(b) = v_begin(b-1) + (vdot_begin(b-1) + vdot_end(b-1)).*0.5.*delta_t;
w_begin(b) = w_begin(b-1) + (wdot_begin(b-1) + wdot_end(b-1)).*0.5.*delta_t;

% Tait-Bryan angles at start of time step
thetaX_begin(b) = thetaX_begin(b-1) + (thetaXdot_begin(b-1)+thetaXdot_end(b-1)).*0.5.*delta_t;
thetaY_begin(b) = thetaY_begin(b-1) + (thetaYdot_begin(b-1)+thetaYdot_end(b-1)).*0.5.*delta_t;
thetaZ_begin(b) = thetaZ_begin(b-1) + (thetaZdot_begin(b-1)+thetaZdot_end(b-1)).*0.5.*delta_t;

% Angular velocities at start of time step
omegaX_begin(b) = omegaX_begin(b-1) + (omegaXdot_begin(b-1)+omegaXdot_end(b-1)).*0.5.*delta_t;
omegaY_begin(b) = omegaY_begin(b-1) + (omegaYdot_begin(b-1)+omegaYdot_end(b-1)).*0.5.*delta_t;
omegaZ_begin(b) = omegaZ_begin(b-1) + (omegaZdot_begin(b-1)+omegaZdot_end(b-1)).*0.5.*delta_t;

% Rate of change of Tait-Bryan angles at start of time step
thetaZdot_begin(b) = omegaY_begin(b).*sin(thetaX_begin(b))+omegaZ_begin(b).*cos(thetaX_begin(b));
if abs(cos(thetaZ_begin(b))) < (omegaY_begin(b).*cos(thetaX_begin(b))-omegaZ_begin(b).*sin(thetaX_begin(b))).*delta_t/pi
    thetaYdot_begin(b) = pi./delta_t;
else
    thetaYdot_begin(b) = (omegaY_begin(b).*cos(thetaX_begin(b))-omegaZ_begin(b).*sin(thetaX_begin(b)))./cos(thetaZ_begin(b));
end;

thetaXdot_begin(b) = omegaX_begin(b) - thetaYdot_begin(b).*sin(thetaZ_begin(b));
end;
% Debris velocities relative to wind at start of time step
UXG_begin = u_begin(b) - windSpeed;
UYG_begin = v_begin(b);
U2G_begin = w_begin(b);

% Relative velocity along principal axes at start of time step
UXP_begin = UXG_begin.*cos(thetaY_begin(b)).*cos(thetaZ_begin(b)) + UYG_begin.*sin(thetaZ_begin(b)) - U2G_begin.*sin(thetaY_begin(b)).*cos(thetaZ_begin(b));

UYP_begin = UXG_begin.*(sin(thetaY_begin(b)).*sin(thetaX_begin(b)) - cos(thetaY_begin(b)).*sin(thetaZ_begin(b)).*cos(thetaX_begin(b))) + UYG_begin.*(cos(thetaZ_begin(b)).*cos(thetaX_begin(b))) + U2G_begin.*(cos(thetaY_begin(b)).*sin(thetaX_begin(b)) + sin(thetaY_begin(b)).*sin(thetaZ_begin(b)).*cos(thetaX_begin(b)));

U2P_begin = UXG_begin.*(sin(thetaY_begin(b)).*cos(thetaX_begin(b)) + cos(thetaY_begin(b)).*sin(thetaZ_begin(b)).*sin(thetaX_begin(b))) - UYG_begin.*(cos(thetaZ_begin(b)).*sin(thetaX_begin(b))) + U2G_begin.*(cos(thetaY_begin(b)).*cos(thetaX_begin(b)) - sin(thetaY_begin(b)).*sin(thetaZ_begin(b)).*sin(thetaX_begin(b)));

URP_begin = sqrt(UXP_begin.^2 + UYP_begin.^2 + U2P_begin.^2);

% Relative velocity ratio at the start of time step
UXP_over_URP_begin = UXP_begin./URP_begin;

%% Calculate Epsilon (Angle of attack) at Start of Time Step
% Calculation of epsilon based on Richards et al. 2008
epsilon_beginMu = asin(UXP_over_URP_begin);
epsilon_begin(b) = normrnd(epsilon_beginMu,...
abs(epsilon_beginMu.*eps_COV));

% Constrain epsilon from -pi/2 to pi/2
epsilon_begin(b) = mod(epsilon_begin(b),pi);
if epsilon_begin(b) > pi/2
    epsilon_begin(b) = epsilon_begin(b) - pi;
elseif epsilon_begin(b) < -pi/2
    epsilon_begin(b) = epsilon_begin(b) + pi;
end;

%% Calculate Gamma (Tilt angle) at Start of Time Step
% Calculation of gamma based on Richards et al. 2008
if abs(U2P_begin./UYP_begin) < 1e-10
    if UYP_begin > 0
        gamma_beginMu = pi./2;
    end;\n
% Debris velocities relative to wind at start of time step
UXG_begin = u_begin(b) - windSpeed;
UYG_begin = v_begin(b);
U2G_begin = w_begin(b);

% Relative velocity along principal axes at start of time step
UXP_begin = UXG_begin.*cos(thetaY_begin(b)).*cos(thetaZ_begin(b)) + UYG_begin.*sin(thetaZ_begin(b)) - U2G_begin.*sin(thetaY_begin(b)).*cos(thetaZ_begin(b));

UYP_begin = UXG_begin.*(sin(thetaY_begin(b)).*sin(thetaX_begin(b)) - cos(thetaY_begin(b)).*sin(thetaZ_begin(b)).*cos(thetaX_begin(b))) + UYG_begin.*(cos(thetaZ_begin(b)).*cos(thetaX_begin(b))) + U2G_begin.*(cos(thetaY_begin(b)).*sin(thetaX_begin(b)) + sin(thetaY_begin(b)).*sin(thetaZ_begin(b)).*cos(thetaX_begin(b)));

U2P_begin = UXG_begin.*(sin(thetaY_begin(b)).*cos(thetaX_begin(b)) + cos(thetaY_begin(b)).*sin(thetaZ_begin(b)).*sin(thetaX_begin(b))) - UYG_begin.*(cos(thetaZ_begin(b)).*sin(thetaX_begin(b))) + U2G_begin.*(cos(thetaY_begin(b)).*cos(thetaX_begin(b)) - sin(thetaY_begin(b)).*sin(thetaZ_begin(b)).*sin(thetaX_begin(b)));

URP_begin = sqrt(UXP_begin.^2 + UYP_begin.^2 + U2P_begin.^2);

% Relative velocity ratio at the start of time step
UXP_over_URP_begin = UXP_begin./URP_begin;

%% Calculate Epsilon (Angle of attack) at Start of Time Step
% Calculation of epsilon based on Richards et al. 2008
epsilon_beginMu = asin(UXP_over_URP_begin);
epsilon_begin(b) = normrnd(epsilon_beginMu,...
abs(epsilon_beginMu.*eps_COV));

% Constrain epsilon from -pi/2 to pi/2
epsilon_begin(b) = mod(epsilon_begin(b),pi);
if epsilon_begin(b) > pi/2
    epsilon_begin(b) = epsilon_begin(b) - pi;
elseif epsilon_begin(b) < -pi/2
    epsilon_begin(b) = epsilon_begin(b) + pi;
end;

%% Calculate Gamma (Tilt angle) at Start of Time Step
% Calculation of gamma based on Richards et al. 2008
if abs(U2P_begin./UYP_begin) < 1e-10
    if UYP_begin > 0
        gamma_beginMu = pi./2;
else
gamma_beginMu = 3.*pi./2;
end;
else
  if UZP_begin > 0
    if atan(UYP_begin./(UZP_begin + 1e-10)) > 0
      gamma_beginMu = atan(UYP_begin./(UZP_begin+1e-10));
    else
      gamma_beginMu = atan(UYP_begin./(UZP_begin+1e-10)) + 2.*pi;
    end;
  else
    gamma_beginMu = atan(UYP_begin./(UZP_begin - 1e-10)) + pi;
  end;
end;

gamma_begin(b) = normrnd(gamma_beginMu,abs(gamma_beginMu.*gam_COV));

% Constrain Gamma from 0 to 2pi
gamma_begin(b) = mod(gamma_begin(b),2*pi);

%% Calculate Debris Forces at Start of Time Step
% Dynamic pressure at start of time step
dyn_press_begin = 0.5.*rhoAir.* URP_begin.^2; % (N/m^2)

% Center of pressure at start of time step
CoP_begin = (Ly.*Lz)./(Lz.*abs(sin(gamma_begin(b))) + Ly.*abs(cos(gamma_begin(b)))); % (m)

% Effective aspect ratio at start of time step
aspectRatio_begin = (Ly.*Lz)./CoP_begin.^2;

% Calculate apparent camber at start of time step
if camber == 1 && b > 1
  appCamber_begin = 2.*pi./(1+(2./aspectRatio_begin)).*max(min(-(epsilon_begin(b)-epsilon_begin(b-1))./delta_t.*CoP_begin.*cos(epsilon_begin(b))./2./URP_begin,...
  max_camber),-max_camber);
else
  appCamber_begin = 0;
end;

% Aerodynamic coefficients at start of time step
% Determine the force coefficient along the 'X_P' axis
CFX_begin = interp2(gamma_array,epsilon_array,Forcex,...
  gamma_begin(b),epsilon_begin(b));
% Include stall hysteresis
if stall == 1 && b > 1 && (abs(epsilon_begin(b)) <
  abs(epsilon_begin(b-1)))
  CFX_hyst_begin = min(abs(CFX_begin),Forcex(1,1)).*...
  CFX_begin./abs(CFX_begin);
else
  CFX_hyst_begin = CFX_begin;
end;

% Prevent a divide by zero within My and Mz
if CFX_hyst_begin == 0
  CFX_hyst_begin = 1e-10;
end;

% Calculate total CFX_begin
CFX_begin = CFX_hyst_begin + appCamber_begin;

% Determine the force coefficient along the 'Y_P' axis
CFY_begin = interp2(gamma_array,epsilon_array,Forcey,...
  gamma_begin(b),epsilon_begin(b));

% Determine the force coefficient along the 'Z_P' axis
CFZ_begin = interp2(gamma_array,epsilon_array,Forcez,...
  gamma_begin(b),epsilon_begin(b));

% Determine the moment coefficient about the 'X_P' axis
CMX_begin = interp2(gamma_array,epsilon_array,Forcemx,...
  gamma_begin(b),epsilon_begin(b));

% Determine the moment coefficient about the 'Y_P' axis
CMY_begin = interp2(gamma_array,epsilon_array,Forcemy,...
  gamma_begin(b),epsilon_begin(b));

% Determine the moment coefficient about the 'Z_P' axis
CMZ_begin = interp2(gamma_array,epsilon_array,Forcemz,...
  gamma_begin(b),epsilon_begin(b));

% Calculate forces along the debris axes at start of time step
FXP_begin = CFX_begin.* dyn_press_begin.*Ax;
FYP_begin = CFY_begin.* dyn_press_begin.*Ay;
FZP_begin = CFZ_begin.* dyn_press_begin.*Az;

% Calculate forces along ground axes at start of time step
FXG_begin = FXP_begin.*cos(thetaZ_begin(b)).*...
  cos(thetaY_begin(b))+FYP_begin.*(sin(thetaX_begin(b)).*...
  sin(thetaY_begin(b)) - cos(thetaX_begin(b))).*...
  sin(thetaZ_begin(b)).*cos(thetaY_begin(b)))+FZP_begin.*...
  (cos(thetaX_begin(b)).*sin(thetaY_begin(b))+...
  sin(thetaX_begin(b)).*sin(thetaZ_begin(b))).*...
  cos(thetaY_begin(b)));

FYG_begin = FXP_begin.*sin(thetaZ_begin(b)) + FYP_begin.*...
\begin{align*}
\cos(\theta_X_{\text{begin}}(b)) \cdot \cos(\theta_Z_{\text{begin}}(b)) - F_{ZP_{\text{begin}}} \cdot \text{mass} \cdot g; \\
F_{ZG_{\text{begin}}} = -F_{XP_{\text{begin}}} \cdot \cos(\theta_Z_{\text{begin}}(b)) \cdot \\
\sin(\theta_Y_{\text{begin}}(b)) + F_{YP_{\text{begin}}} \cdot (\cos(\theta_X_{\text{begin}}(b)) \cdot \\
\sin(\theta_Y_{\text{begin}}(b)) \cdot \sin(\theta_Z_{\text{begin}}(b)) + \\
\cos(\theta_X_{\text{begin}}(b)) \cdot \sin(\theta_Y_{\text{begin}}(b)) + F_{ZP_{\text{begin}}}) \cdot \\
\cos(\theta_Y_{\text{begin}}(b)) \cdot \sin(\theta_Z_{\text{begin}}(b)) - \\
\sin(\theta_X_{\text{begin}}(b)) \cdot \sin(\theta_Y_{\text{begin}}(b)) \cdot \\
\sin(\theta_Z_{\text{begin}}(b));
\end{align*}

\% Calculate accelerations of debris at start of time step
udot_{\text{begin}}(b) = F_{XG_{\text{begin}}}/\text{mass};
\vdot_{\text{begin}}(b) = F_{YG_{\text{begin}}}/\text{mass};
\wdot_{\text{begin}}(b) = F_{ZG_{\text{begin}}}/\text{mass};

\% Calculate moments about principal axes at start of time step
M_x_{\text{begin}} = C_{MX_{\text{begin}}} \cdot \text{dyn\_press\_begin} \cdot (L_z \cdot A_y + L_y \cdot A_z) + \\
\hspace{1cm} C_{DMX} \cdot 0.5 \cdot \rho_{\text{Air}} \cdot (\text{abs}(U_{RP_{\text{begin}}}) + \text{abs}(\omega_{\text{X_{\text{begin}}}(b)}) \cdot \\
\hspace{1cm} 0.5 \cdot \sqrt{(L_y)^2 + (L_z)^2}) \cdot (L_z \cdot A_y + L_y \cdot A_z) \cdot \\
\hspace{1.5cm} \omega_{\text{X_{\text{begin}}}(b)} \cdot \sqrt{(L_y)^2 + (L_z)^2};

M_y_{\text{begin}} = C_{MY_{\text{begin}}} \cdot \text{dyn\_press\_begin} \cdot (L_z \cdot A_x + L_y \cdot A_z) \cdot \\
\hspace{1cm} \text{(CFX_{\text{begin}} \cdot CFX_{\text{hyst\_begin}})} + C_{DMY} \cdot 0.5 \cdot \rho_{\text{Air}} \cdot \hspace{1cm} \\
\hspace{1.0cm} (\text{abs}(U_{RP_{\text{begin}}}) + \text{abs}(\omega_{\text{Y_{\text{begin}}}(b)} \cdot \\
\hspace{1.5cm} 0.5 \cdot \sqrt{(L_x)^2 + (L_z)^2}) \cdot (L_z \cdot A_x + L_y \cdot A_z) \cdot \omega_{\text{Y_{\text{begin}}}(b)} \cdot \\
\hspace{2.0cm} \sqrt{(L_x)^2 + (L_z)^2};

M_z_{\text{begin}} = C_{MZ_{\text{begin}}} \cdot \text{dyn\_press\_begin} \cdot (L_x \cdot A_x + L_x \cdot A_y) \cdot \\
\hspace{1cm} \text{(CFX_{\text{begin}} \cdot CFX_{\text{hyst\_begin}})} + C_{DMZ} \cdot 0.5 \cdot \rho_{\text{Air}} \cdot \hspace{1cm} \\
\hspace{1.0cm} (\text{abs}(U_{RP_{\text{begin}}}) + \text{abs}(\omega_{\text{Z_{\text{begin}}}(b)} \cdot \\
\hspace{1.5cm} 0.5 \cdot \sqrt{(L_x)^2 + (L_y)^2}) \cdot (L_x \cdot A_x + L_x \cdot A_y) \cdot \omega_{\text{Z_{\text{begin}}}(b)} \cdot \\
\hspace{2.0cm} \sqrt{(L_x)^2 + (L_y)^2};

\% Calculate angular accelerations at start of time step
omega_{X_{\text{dot\_begin}}}(b) = M_{X_{\text{begin}}} / I_{xx} - (I_{zz} - I_{yy}) \cdot \omega_{\text{Y_{\text{begin}}}(b)} \cdot \\
\hspace{2cm} (omega_{Z_{\text{begin}}}(b) / I_{xx});
\omega_{Y_{\text{dot\_begin}}}(b) = (M_{Y_{\text{begin}}} - (I_{xx} - I_{zz}) \cdot \omega_{\text{X_{\text{begin}}}(b)} \cdot \omega_{\text{Z_{\text{begin}}}(b)}) / I_{yy};
\omega_{Z_{\text{dot\_begin}}}(b) = (M_{Z_{\text{begin}}} - (I_{yy} - I_{xx}) \cdot \omega_{\text{X_{\text{begin}}}(b)} \cdot \omega_{\text{Y_{\text{begin}}}(b)}) / I_{zz};

\% End simulation when vertical displacement <= 0 meters
if y_{\text{begin}}(b) <= 0
  break;
end;

\% Calculate Values for the End of Time Step
\% Debris velocities at end of time step
u_{\text{end}}(b) = u_{\text{begin}}(b) + u_{\text{dot\_begin}}(b) \cdot \text{delta\_t};
v_{\text{end}}(b) = v_{\text{begin}}(b) + v_{\text{dot\_begin}}(b) \cdot \text{delta\_t};
w_{\text{end}}(b) = w_{\text{begin}}(b) + w_{\text{dot\_begin}}(b) \cdot \text{delta\_t};
% Debris velocities relative to wind at end of time step
UXG_end = u_end(b) - windSpeed;
UYG_end = v_end(b);
UZG_end = w_end(b);

% Tait-Bryan angles at end of time step
thetaX_end = thetaX_begin(b) + thetaXdot_begin(b).*delta_t;
thetaY_end = thetaY_begin(b) + thetaYdot_begin(b).*delta_t;
thetaZ_end = thetaZ_begin(b) + thetaZdot_begin(b).*delta_t;

% Angular velocities at end of time step
omegaX_end = omegaX_begin(b) + omegaXdot_begin(b).*delta_t;
omegaY_end = omegaY_begin(b) + omegaYdot_begin(b).*delta_t;
omegaZ_end = omegaZ_begin(b) + omegaZdot_begin(b).*delta_t;

% Rate of change of Tait-Bryan angles at end of time step
thetaZdot_end(b) = omegaY_end.*sin(thetaX_end) + ... 
omegaZ_end.*cos(thetaX_end);
if abs(cos(thetaZ_end)) < (omegaY_end.*cos(thetaX_end)-... 
omegaZ_end.*sin(thetaX_end)).*delta_t/pi
    thetaYdot_end(b) = pi./delta_t;
else
    thetaYdot_end(b) = (omegaY_end.*cos(thetaX_end)-... 
omegaZ_end.*sin(thetaX_end))./cos(thetaZ_end);
end;

thetaXdot_end(b) = omegaX_end - thetaYdot_end(b).* ... 
sin(thetaZ_end);

% Relative velocities along principal axes at end of time step
UXP_end = UXG_end.*cos(thetaY_end).*cos(thetaZ_end)+UYG_end... 
.*sin(thetaZ_end)-UZG_end.*sin(thetaY_end).* ... 
cos(thetaZ_end);
UYP_end = UXG_end.*(sin(thetaY_end).*sin(thetaX_end)-... 
cos(thetaY_end).*sin(thetaZ_end).*cos(thetaX_end))+... 
UYG_end.*(cos(thetaZ_end).*cos(thetaX_end)).*cos(thetaZ_end)) +
(UYG_end.*cos(thetaZ_end).*sin(thetaX_end) +sin(thetaY_end).* ... 
sin(thetaZ_end).*cos(thetaX_end));
UZP_end = UXG_end.*(sin(thetaY_end).*cos(thetaX_end)+... 
cos(thetaY_end).*sin(thetaZ_end).*sin(thetaX_end)) - ... 
UYG_end.*(cos(thetaZ_end).*sin(thetaX_end)) +UZG_end.* ... 
(cos(thetaY_end).*cos(thetaX_end) -sin(thetaY_end).* ... 
sin(thetaZ_end).*sin(thetaX_end));
URP_end = sqrt(UXP_end.^2 + UYP_end.^2 + UZP_end.^2);

% Relative velocity ratio at the end of the time step
UXP_over_URP_end = UXP_end./URP_end;
%% Calculate Epsilon (Angle of attack) at End of Time Step
% Calculation of epsilon based on Richards et al. 2008
epsilon_endMu = asin(UXP_over_URP_end);
epsilon_end(b) = normrnd(epsilon_endMu,abs(epsilon_endMu.*... 
eps_COV));

% The deterministic model limits epsilon from pi/2 to -pi/2
% Since there is a small possibility that we can obtain an
% angle for epsilon either > pi/2 or < -pi/2, we must
% constrain the angles at these points since an epsilon value >
% pi/2 or < -pi/2 would imply that debris is flying into the
% wind, which is physically impossible in a uniform horizontal
% wind speed based on the definition of epsilon.
if epsilon_end(b) > pi/2
    epsilon_end(b) = pi/2;
elseif epsilon_end(b) < -pi/2
    epsilon_end(b) = -pi/2;
end;

%% Calculate Gamma (Tilt Angle) at End of Time Step
% Calculation of gamma based on Richards et al. 2008
if abs(UZP_end./UYP_end) < 1e-10
    if UYP_end > 0
        gamma_endMu = pi./2;
    else
        gamma_endMu = 3.*pi./2;
    end;
else
    if atan(UYP_end./(UZP_end + 1e-10)) > 0
        gamma_endMu = atan(UYP_end./(UZP_end + 1e-10));
    else
        gamma_endMu = atan(UYP_end./(UZP_end + 1e-10)) +...
                     2.*pi;
    end;
else
    gamma_endMu = atan(UYP_end./(UZP_end - 1e-10)) + pi;
end;

gamma_end(b) = normrnd(gamma_endMu,abs(gamma_endMu.*gam_COV));

% Constrain Gamma from 0 to 2pi
gamma_end(b) = mod(gamma_end(b),2*pi);

%% Calculate Debris Forces at End of Time Step
% Dynamic pressure at end of time step
dyn_press_end = 0.5.*rhoAir.* URP_end.^2; % (N/m^2)
% Center of pressure at end of time step
CoP_end = (Ly.*Lz)./(Lz.*abs(sin(gamma_end(b))) + Ly.*...  
    abs(cos(gamma_end(b)))); \% (m)

% Effective aspect ratio at end of time step
aspectRatio_end = (Ly.*Lz)./CoP_end.^2;

% Calculate apparent camber at end of time step
if camber == 1 && b > 1
    appCamber_end = 2.*pi./(1+(2./aspectRatio_end)).*...  
        max(min(-(epsilon_end(b)-epsilon_end(b1))./delta_t.*...  
            CoP_end.*cos(epsilon_end(b))./2./URP_end,...  
            max_camber),-max_camber);
else
    appCamber_end = 0;
end;

% Aerodynamic coefficients at end of time step
% Determine the force coefficient along the 'X_P' axis
CFX_end = interp2(gamma_array,epsilon_array,Forcex,...  
    gamma_end(b),epsilon_end(b));

% Include stall hysteresis
if stall == 1 && b > 1 && (abs(epsilon_end(b)) < ...  
    abs(epsilon_end(b-1))
    CFX_hyst_end = min(abs(CFX_end),Forcex(1,1)).*...  
        CFX_end./abs(CFX_end);
else
    CFX_hyst_end = CFX_end;
end;

% Prevent a divide by zero within My and Mz
if CFX_hyst_end == 0
    CFX_hyst_end = 1e-17;
end;

% Calculate total CFX_end
CFX_end = CFX_hyst_end + appCamber_end;

% Determine the force coefficient along the 'Y_P' axis
CFY_end = interp2(gamma_array,epsilon_array,Forcey,...  
    gamma_end(b),epsilon_end(b));

% Determine the force coefficient along the 'Z_P' axis
CFZ_end = interp2(gamma_array,epsilon_array,Forcez,...  
    gamma_end(b),epsilon_end(b));

% Determine the moment coefficient about the 'X_P' axis
CMX_end = interp2(gamma_array,epsilon_array,Forcemx,...  
    gamma_end(b),epsilon_end(b));
% Determine the moment coefficient about the 'Y_P' axis
CMY_end = interp2(gamma_array,epsilon_array,Forcemy,...
    gamma_end(b),epsilon_end(b));

% Determine the moment coefficient about the 'Z_P' axis
CMZ_end = interp2(gamma_array,epsilon_array,Forcemz,...
    gamma_end(b),epsilon_end(b));

% Calculate forces along the debris axes at end of time step
FXP_end = CFX_end.* dyn_press_end.*Ax;
FYP_end = CFY_end.*dyn_press_end.*Ay;
FZP_end = CFZ_end.*dyn_press_end.*Az;

% Calculate force along ground fixed axes at end of time step
FXG_end = FXP_end.*cos(thetaZ_end).*cos(thetaY_end)+...
    FYP_end.*(sin(thetaX_end).*sin(thetaY_end)- ...
    cos(thetaX_end).*sin(thetaZ_end).*cos(thetaY_end))+...
    FZP_end.*(cos(thetaX_end).*sin(thetaY_end)+...
    sin(thetaX_end).*sin(thetaZ_end).*cos(thetaY_end));

FYG_end = FXP_end.*sin(thetaZ_end) + FYP_end.*...
    cos(thetaX_end).*sin(thetaZ_end) - FZP_end.*...
    sin(thetaX_end).*cos(thetaZ_end)-mass.*g;

FZG_end = -FXP_end.*cos(thetaZ_end).*sin(thetaY_end)+...
    FYP_end.*(cos(thetaX_end).*sin(thetaY_end).*...
    sin(thetaX_end)+sin(thetaX_end).*cos(thetaY_end))+...
    FZP_end.*(cos(thetaX_end).*cos(thetaY_end)-...
    sin(thetaX_end).*sin(thetaZ_end).*sin(thetaY_end));

% Calculate acceleration of debris at end of time step
udot_end(b) = FXG_end./mass;
vdot_end(b) = FYG_end./mass;
wdot_end(b) = FZG_end./mass;

% Calculate moments about principal axes at end of time step
Mx_end = CMX_end.*dyn_press_end.*(Lz.*Ay+Ly.*Az)+CDMX.*0.5.*...
    rhoAir.*(abs(URP_end)+abs(omegaX_end).*0.5.*sqrt(Ly.^2+Lz.^2)).*(Lz.*Ay+Ly.*Az).*omegaX_end.*...
    sqrt(Ly.^2+Lz.^2);  

My_end = CMY_end.*dyn_press_end.*(Lz.*Ax+Lx.*Az).*...
    (CFX_end./CFX_hyst_end)+CDMY.*0.5.*rhoAir .*...
    (abs(URP_end)+ abs(omegaY_end).* 0.*sqrt(Lx.^2+Lz.^2)).*...
    (Lz.*Ax+Lx.*Az).* omegaY_end .* sqrt(Lx.^2+Lz.^2);  

Mz_end = CMZ_end.*dyn_press_end.*(Ly.*Ax+Lx.*Ay).*...
    (CFX_end./CFX_hyst_end)+CDMZ .* 0.5 .* rhoAir .*...
    (abs(URP_end)+abs(omega2_end).*0.5.*sqrt(Lx.^2+Ly.^2)).*...
    (Ly.*Ax+Lx.*Ay).* omega2_end .* sqrt(Lx.^2+Ly.^2);
% Calculate angular accelerations at end of time step
omegaXdot_end(b) = Mx_end./Ixx - (Izz - Iyy).*omegaY_end.*
(omegaZ_end./Ixx);
omegaYdot_end(b) = (My_end - (Ixx - Izz).*omegaX_end.*
omegaZ_end)./Iyy;
omegaZdot_end(b) = (Mz_end - (Iyy - Ixx).*omegaX_end.*
omegaY_end)./Izz;
end;

%% Truncate End Zeros from Stored Variables
x = x_begin(1:b); % Longitudinal position (m)
y = y_begin(1:b); % Vertical position (m)
z = z_begin(1:b); % Lateral position (m)
u = u_begin(1:b); % Longitudinal velocity (m/s)
v = v_begin(1:b); % Vertical velocity (m/s)
w = w_begin(1:b); % Lateral velocity (m/s)

% Tait-Bryan angles defined in Figure 3.1
roll = thetaX_begin(1:b); % (rad)
yaw = thetaY_begin(1:b); % (rad)
pitch = thetaZ_begin(1:b); % (rad)

p = omegaX_begin(1:b); % (rad/s)
q = omegaY_begin(1:b); % (rad/s)
r = omegaZ_begin(1:b); % (rad/s)
pdot = omegaXdot_begin(1:b); % (rad/s^2)
qdot = omegaYdot_begin(1:b); % (rad/s^2)
rdot = omegaZdot_begin(1:b); % (rad/s^2)

%% Interpolate to Find Final Trajectory Values
% Interpolate trajectory values for vertical position (y) = 0
x(end) = interp1(y, x, 0);
z(end) = interp1(y, z, 0);
u(end) = interp1(y, u, 0);
v(end) = interp1(y, v, 0);
w(end) = interp1(y, w, 0);
roll(end) = interp1(y, roll, 0);
yaw(end) = interp1(y, yaw, 0);
pitch(end) = interp1(y, pitch, 0);
p(end) = interp1(y, p, 0);
q(end) = interp1(y, q, 0);
r(end) = interp1(y, r, 0);
pdot(end) = interp1(y, pdot, 0);
qdot(end) = interp1(y, qdot, 0);
rdot(end) = interp1(y, rdot, 0);
y(end) = 0; % Set final vertical value to zero
%% Write Output to Text File
fid = fopen([num2str(Lx),'x',num2str(Ly),'x',num2str(Lz),...  
    debrisIdentifier,'_Wind',num2str(windSpeed),...
    '_Sim',num2str(a),'.txt'], 'a');
header = ['  x(m)            y(m)             z(m)          ',...
            'u(m/s)           v(m/s)           w(m/s)         ',...
            'roll(rad)       yaw(rad)       pitch(rad)       ',...
            'p(rad/s)        q(rad/s)        r(rad/s)      ',...
            'pdot(rad/s2)    qdot(rad/s2)    rdot(rad/s2)'];
fprintf(fid,'%s',header);
dataRep = ['
',repmat('%+10.6f		',1,15),];
fprintf(fid,dataRep,[x;y;z;u;v;w;roll;yaw;pitch;p;q;r;pdot;...
    qdot;rdot]);
fclose(fid);

end;
B.2: Short Fourier Series Coefficients Code

% Short fourier series coefficients based on Richards 2010
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%
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%
% Data from deterministic debris trajectory model provided by:
% Dr. Peter J. Richards
% Department of Mechanical Engineering
% University of Auckland
%
% Last Modified: Nov-25-2011
% by: J. Michael Grayson
%
% Declare Function
function [Fourier_fx, Fourier_fy, Fourier_fz, Fourier_mx, Fourier_my, ...
    Fourier_mz] = Fourier_Series(shapeIndex)
% Assign Short Fourier Series Coefficients to Appropriate Debris Type
% Load .mat file containing short Fourier series coefficients
load Short_Fourier.mat;

if shapeIndex == 1
    fourier_fx = Short_Fourier(1:10,1:7);
    fourier_fy = Short_Fourier(12:21,1:6);
    fourier_fz = Short_Fourier(23:32,1:6);
    fourier_mx = Short_Fourier(1:10,9:14);
    fourier_my = Short_Fourier(12:21,9:14);
    fourier_mz = Short_Fourier(23:32,9:14);
elseif shapeIndex == 2
    fourier_fx = Short_Fourier(1:10,16:22);
    fourier_fy = Short_Fourier(12:21,16:21);
    fourier_fz = Short_Fourier(23:32,16:21);
    fourier_mx = Short_Fourier(1:10,24:29);
    fourier_my = Short_Fourier(12:21,24:29);
    fourier_mz = Short_Fourier(23:32,24:29);
elseif shapeIndex == 3
    fourier_fx = Short_Fourier(1:10,31:37);
    fourier_fy = Short_Fourier(12:21,31:36);
fourier_fz = Short_Fourier(23:32,31:36);
fourier_mx = Short_Fourier(1:10,39:44);
fourier_my = Short_Fourier(12:21,39:44);
fourier_mz = Short_Fourier(23:32,39:44);

elseif shapeIndex == 4
    fourier_fx = Short_Fourier(1:10,46:52);
fourier_fy = Short_Fourier(12:21,46:51);
fourier_fz = Short_Fourier(23:32,46:51);
fourier_mx = Short_Fourier(1:10,54:59);
fourier_my = Short_Fourier(12:21,54:59);
fourier_mz = Short_Fourier(23:32,54:59);
elseif shapeIndex == 5
    fourier_fx = Short_Fourier(1:10,61:67);
fourier_fy = Short_Fourier(12:21,61:66);
fourier_fz = Short_Fourier(23:32,61:66);
fourier_mx = Short_Fourier(1:10,69:74);
fourier_my = Short_Fourier(12:21,69:74);
fourier_mz = Short_Fourier(23:32,69:74);
elseif shapeIndex == 6
    fourier_fx = Short_Fourier(1:10,76:82);
fourier_fy = Short_Fourier(12:21,76:81);
fourier_fz = Short_Fourier(23:32,76:81);
fourier_mx = Short_Fourier(1:10,84:89);
fourier_my = Short_Fourier(12:21,84:89);
fourier_mz = Short_Fourier(23:32,84:89);
end;

%% Create Total Short Fourier Series Coefficients Matrix
% Populate matrices with zeros
Fourier_fx = zeros(30,25);
Fourier_fy = zeros(30,25);
Fourier_fz = zeros(30,25);
Fourier_mx = zeros(30,25);
Fourier_my = zeros(30,25);
Fourier_mz = zeros(30,25);

% Assign coefficients to total matrix at appropriate locations
Fourier_fx(1:10,19:25) = fourier_fx;
Fourier_fy(11:20,1:6) = fourier_fy;
Fourier_fz(11:20,7:12) = fourier_fz;
Fourier_mx(11:20,13:18) = fourier_mx;
Fourier_mz(21:30,1:6) = fourier_mz;
Fourier_my(21:30,7:12) = fourier_my;
B.3: Aerodynamic Force and Moment Coefficients Code

% Uses bi-directional short fourier series proposed by Richards 2010 to
% create an array of aerodynamic force and moment coefficients based on
% the value of the angle of attack (epsilon) and the tilt angle (gamma)
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%
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%
% Data obtained from deterministic debris trajectory model provided by:
% Dr. Peter J. Richards
% Department of Mechanical Engineering
% University of Auckland
%
% Last Modified: Nov-25-2011
% by: J. Michael Grayson

% Declare Function
function [Forcex, Forcey, Forcez, Forcemx, Forcemy, Forcemz] = ...
    aeroCoeff(Fourier_fx, Fourier_fy, Fourier_fz, Fourier_mx,...
                Fourier_my, Fourier_mz, gamma, epsilon)
% Calculate Aerodynamic Force and Moment Coefficients
% Preallocation
intermMatrix = zeros(size(Fourier_fx,1), length(gamma),6);
forceMatrix = zeros(length(epsilon),length(gamma), 6);

% Calculate intermediate matrix for 'X' force coefficients
for a = 1:6 % 6 = number of force and moment coefficients
    if a == 1
        fourierCoeff = Fourier_fx;
    elseif a == 2
        fourierCoeff = Fourier_fy;
    elseif a == 3
        fourierCoeff = Fourier_fz;
    elseif a == 4
        fourierCoeff = Fourier_mx;
    elseif a == 5
        fourierCoeff = Fourier_my;
    elseif a == 6
        fourierCoeff = Fourier_mz;
    end;
for b = 1:size(fourierCoeff,1)
    for c = 1:length(gamma) %Short Fourier series for gamma
        intermMatrix(b,c,a) = fourierCoeff(b,1).*sin(gamma(c)) +
            fourierCoeff(b,2).*sin(3.*gamma(c)) +
            fourierCoeff(b,3).*sin(5.*gamma(c)) +
            fourierCoeff(b,4).*sin(7.*gamma(c)) +
            fourierCoeff(b,5).*sin(9.*gamma(c)) +
            fourierCoeff(b,6).*sin(11.*gamma(c)) +
            fourierCoeff(b,7).*cos(gamma(c)) +fourierCoeff(b,8).*
            cos(3.*gamma(c)) +fourierCoeff(b,9).*
            cos(5.*gamma(c)) +fourierCoeff(b,10).*
            cos(7.*gamma(c)) +fourierCoeff(b,11).*
            cos(9.*gamma(c)) +fourierCoeff(b,12).*
            cos(11.*gamma(c)) +fourierCoeff(b,13).*
            cos(13.*gamma(c)) +fourierCoeff(b,14).*
            cos(15.*gamma(c)) +fourierCoeff(b,16).*
            cos(17.*gamma(c)) +fourierCoeff(b,17).*
            cos(19.*gamma(c)) +fourierCoeff(b,18).*
            cos(21.*gamma(c)) +fourierCoeff(b,19).*
            cos(23.*gamma(c)) +fourierCoeff(b,20).*
            cos(25.*gamma(c)) +fourierCoeff(b,21).*
            cos(27.*gamma(c)) +fourierCoeff(b,22).*
            cos(29.*gamma(c)) +fourierCoeff(b,23).*
            cos(31.*gamma(c)) +fourierCoeff(b,24).*
            cos(33.*gamma(c)) +fourierCoeff(b,25).*
            cos(35.*gamma(c)) +fourierCoeff(b,26).*
            cos(37.*gamma(c)) +fourierCoeff(b,27).*
            cos(39.*gamma(c)) +fourierCoeff(b,28).*
            cos(41.*gamma(c)) +fourierCoeff(b,29).*
            cos(43.*gamma(c)) +fourierCoeff(b,30).*
            cos(45.*gamma(c)) +fourierCoeff(b,31).*
            cos(47.*gamma(c)) +fourierCoeff(b,32).*
            cos(49.*gamma(c)) +fourierCoeff(b,33).*
            cos(51.*gamma(c)) +fourierCoeff(b,34).*
            cos(53.*gamma(c)) +fourierCoeff(b,35).*
            cos(55.*gamma(c)) +fourierCoeff(b,36).*
            cos(57.*gamma(c)) +fourierCoeff(b,37).*
            cos(59.*gamma(c)) +fourierCoeff(b,38).*
            cos(61.*gamma(c)) +fourierCoeff(b,39).*
            cos(63.*gamma(c)) +fourierCoeff(b,40).*
            cos(65.*gamma(c)) +fourierCoeff(b,41).*
            cos(67.*gamma(c)) +fourierCoeff(b,42).*
            cos(69.*gamma(c)) +fourierCoeff(b,43).*
            cos(71.*gamma(c)) +fourierCoeff(b,44).*
            cos(73.*gamma(c)) +fourierCoeff(b,45).*
            cos(75.*gamma(c)) +fourierCoeff(b,46).*
            cos(77.*gamma(c)) +fourierCoeff(b,47).*
            cos(79.*gamma(c)) +fourierCoeff(b,48).*
            cos(81.*gamma(c)) +fourierCoeff(b,49).*
            cos(83.*gamma(c)) +fourierCoeff(b,50).*
            cos(85.*gamma(c)) +fourierCoeff(b,51).*
            cos(87.*gamma(c)) +fourierCoeff(b,52).*
            cos(89.*gamma(c)) +fourierCoeff(b,53).*
            cos(91.*gamma(c)) +fourierCoeff(b,54).*
            cos(93.*gamma(c)) +fourierCoeff(b,55).*
            cos(95.*gamma(c)) +fourierCoeff(b,56).*
            cos(97.*gamma(c)) +fourierCoeff(b,57).*
            cos(99.*gamma(c)) +fourierCoeff(b,58).*
            cos(101.*gamma(c)) +fourierCoeff(b,59).*
            cos(103.*gamma(c)) +fourierCoeff(b,60).*
            cos(105.*gamma(c)) +fourierCoeff(b,61).*
            cos(107.*gamma(c)) +fourierCoeff(b,62).*
            cos(109.*gamma(c)) +fourierCoeff(b,63).*
            cos(111.*gamma(c)) +fourierCoeff(b,64).*
            cos(113.*gamma(c)) +fourierCoeff(b,65).*
            cos(115.*gamma(c)) +fourierCoeff(b,66).*
            cos(117.*gamma(c)) +fourierCoeff(b,67).*
            cos(119.*gamma(c)) +fourierCoeff(b,68).*
            cos(121.*gamma(c)) +fourierCoeff(b,69).*
            cos(123.*gamma(c)) +fourierCoeff(b,70).*
            cos(125.*gamma(c)) +fourierCoeff(b,71).*
            cos(127.*gamma(c)) +fourierCoeff(b,72).*
            cos(129.*gamma(c)) +fourierCoeff(b,73).*
            cos(131.*gamma(c)) +fourierCoeff(b,74).*
            cos(133.*gamma(c)) +fourierCoeff(b,75).*
            cos(135.*gamma(c)) +fourierCoeff(b,76).*
            cos(137.*gamma(c)) +fourierCoeff(b,77).*
            cos(139.*gamma(c)) +fourierCoeff(b,78).*
            cos(141.*gamma(c)) +fourierCoeff(b,79).*
            cos(143.*gamma(c)) +fourierCoeff(b,80).*
            cos(145.*gamma(c)) +fourierCoeff(b,81).*
            cos(147.*gamma(c)) +fourierCoeff(b,82).*
            cos(149.*gamma(c)) +fourierCoeff(b,83).*
            cos(151.*gamma(c)) +fourierCoeff(b,84).*
            cos(153.*gamma(c)) +fourierCoeff(b,85).*
            cos(155.*gamma(c)) +fourierCoeff(b,86).*
            cos(157.*gamma(c)) +fourierCoeff(b,87).*
            cos(159.*gamma(c)) +fourierCoeff(b,88).*
            cos(161.*gamma(c)) +fourierCoeff(b,89).*
            cos(163.*gamma(c)) +fourierCoeff(b,90).*
            cos(165.*gamma(c)) +fourierCoeff(b,91).*
            cos(167.*gamma(c)) +fourierCoeff(b,92).*
            cos(169.*gamma(c)) +fourierCoeff(b,93).*
            cos(171.*gamma(c)) +fourierCoeff(b,94).*
            cos(173.*gamma(c)) +fourierCoeff(b,95).*
            cos(175.*gamma(c)) +fourierCoeff(b,96).*
            cos(177.*gamma(c)) +fourierCoeff(b,97).*
            cos(179.*gamma(c)) +fourierCoeff(b,98).*
            cos(181.*gamma(c)) +fourierCoeff(b,99).*
            cos(183.*gamma(c)) +fourierCoeff(b,100).*
        end;
    end;
for d = 1:length(epsilon)% Short Fourier series for epsilon
    for e = 1:length(gamma)
        forceMatrix(d,e,a) = intermMatrix(1,e,a).*
            sin(epsilon(d)) +intermMatrix(2,e,a).*
            sin(3.*epsilon(d)) +intermMatrix(3,e,a).*
            sin(5.*epsilon(d)) +intermMatrix(4,e,a).*
            sin(7.*epsilon(d)) +intermMatrix(5,e,a).*
            sin(9.*epsilon(d)) +intermMatrix(6,e,a).*
            sin(11.*epsilon(d)) +intermMatrix(7,e,a).*
            sin(13.*epsilon(d)) +intermMatrix(8,e,a).*
            sin(15.*epsilon(d)) +intermMatrix(9,e,a).*
            sin(17.*epsilon(d)) +intermMatrix(10,e,a).*
            sin(19.*epsilon(d)) +intermMatrix(11,e,a).*
            sin(21.*epsilon(d)) +intermMatrix(12,e,a).*
            sin(23.*epsilon(d)) +intermMatrix(13,e,a).*
            sin(25.*epsilon(d)) +intermMatrix(14,e,a).*
            sin(27.*epsilon(d)) +intermMatrix(15,e,a).*
            sin(29.*epsilon(d)) +intermMatrix(16,e,a).*
            sin(31.*epsilon(d)) +intermMatrix(17,e,a).*
            sin(33.*epsilon(d)) +intermMatrix(18,e,a).*
            sin(35.*epsilon(d)) +intermMatrix(19,e,a).*
            sin(37.*epsilon(d)) +intermMatrix(20,e,a).*
            sin(39.*epsilon(d)) +intermMatrix(21,e,a).*
            sin(41.*epsilon(d)) +intermMatrix(22,e,a).*
            sin(43.*epsilon(d)) +intermMatrix(23,e,a).*
            sin(45.*epsilon(d)) +intermMatrix(24,e,a).*
        end;
    end;
end;
\[ \sin(8.\epsilon(d)) + \text{intermMatrix}(25,e,a) \times \cdots \]
\[ \sin(10.\epsilon(d)) + \text{intermMatrix}(26,e,a) \times \cdots \]
\[ \sin(12.\epsilon(d)) + \text{intermMatrix}(27,e,a) \times \cdots \]
\[ \sin(14.\epsilon(d)) + \text{intermMatrix}(28,e,a) \times \cdots \]
\[ \sin(16.\epsilon(d)) + \text{intermMatrix}(29,e,a) \times \cdots \]
\[ \sin(18.\epsilon(d)) + \text{intermMatrix}(30,e,a) \times \cdots \]
\[ \sin(20.\epsilon(d)) \]

end;
end;
end;

%% Assign Aerodynamic Force and Moment Coefficients To Variables
Forcex = forceMatrix(:,:,1);
Forcey = forceMatrix(:,:,2);
Forcez = forceMatrix(:,:,3);
Forcemx = forceMatrix(:,:,4);
Forcemy = forceMatrix(:,:,5);
Forcemz = forceMatrix(:,:,6);
REFERENCES


