LINEARIZED PERFORMANCE ANALYSIS
OF SAILING AND MOTOR-SAILING

by

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NOMENCLATURE:

AR  aspect ratio: a measure of the slenderness of a wing planform

CL  lift coefficient: see introduction

CD  drag coefficient: see introduction

CDO parasitic drag coefficient: indicates drag caused by rate of loss of linear momentum of a fluid

CDi induced drag coefficient: indicates drag due to tip vortex generation

Cep  effective power coefficient: indicates fuel-saving benefit of rig/keel combination

Cer  effective drive coefficient: indicates net thrust from rig/keel combination

D  distance along track

FH  heeling force from rig

FR  driving force from rig

FS  side force from keel

L  lift

P  constant: indicates power-dependent ship operating costs

Ri  induced resistance: associated with formation of tip vortex on keel

RDk parasitic resistance: associated with rate of loss of linear momentum as the sea flows around the keel

RT  upright resistance: water resistance of upright hull, without keel

SFC specific fuel consumption of engine

Tr  engine power/unit time

Net propeller thrust

V  speed (see section on suffixes also)

a  ratio of ship speed/true wind speed

C  wing chord: the streamwise dimension of a wing
K induced drag factor: a constant in the drag equation - its value depends on the span loading of a wing

pdf probability density of wind strength and direction

x longitudinal axis of a ship

y lateral axis of a ship see Appendix

z vertical axis of a ship

c ship resistance constant: indicates upright ship water resistance

θ angular variable for wind statistical data (=0 North)

V volumetric displacement of a ship

ϕ angle of heel

β apparent wind angle: angle between course sailed and vector sum of true wind velocity and air velocity arising from ship motion -(see Figure 1)

γ true wind angle: angle between course sailed and true wind direction - (See Figure 1)

γ₀ half tacking angle: the true wind angle used when tacking

ρ density: used with suffixes a or s to denote air or sea

ηₜ transmission efficiency from engine to propeller quasi propulsive efficiency: accounts for both

η₅ propeller efficiency and adverse hydrodynamic interaction between propeller and hull

Suffix:

A apparent
A air
A effective
A fuel
A linearized
A power
A drive
A ship or sea (note however that F denotes ship sideforce)
A true
A keel

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1. INTRODUCTION

Analysis of a sailing or motor-sailing vessel is acknowledged to be a difficult problem involving many variables. It is both complex and nonlinear, making it difficult for naval architects to quantify the potential benefits of propelling a ship with some type of rig/keel combination. The aim of this paper is to present a simplified analysis to quantify the benefits of windship propulsion. Simplification is done via nondimensional analysis, separation of slight from extreme nonlinearities and the introduction of new coefficients describing the net propulsive and fuel-saving benefits of a rig/hull/keel combination. Methods are shown that combine statistical weather data with rig/hull/keel data to arrive at an expected voyage time (under sail alone) or expected fuel saving under motor sail (assuming ship speed fixed).

Both sea and air fluid flow problems are based on axes defined relative to the course sailed. This choice of coordinate system eliminates leeway angles as variables in the algebra and implies that "apparent" wind angles are relative to the course sailed and not the ship center line. Conventional definitions of lift and drag coefficients are used in the analysis:

\[ L = \frac{1}{2} \rho V^2 S C_L \]
\[ D = \frac{1}{2} \rho V^2 S C_D \]

where \( L \) is the aerodynamic lift force, \( D \) is the aerodynamic drag force. Other variables are defined under Nomenclature.

Simple wing theory gives the drag coefficient in terms of the lift coefficient.

\[ C_D = C_{DO} + \frac{K C_L^2}{\pi A_R} \]

There are very few areas of powered-ship design which could not be changed by the introduction of some form of wind assistance, and this paper does not presume to rigorously explore every aspect of the problem. The analytical development is deliberately kept simple, both as a means of examining the problem as a whole and because fairly conservative expectations of accuracy are appropriate to any performance calculations involving wind power.

The word "linearized" has a number of implications for both this paper and the wider problem. For this paper, it means that an analysis will be attempted for an essentially linear lifting problem where there is no separation involved in the lifting process. This implies that both rig and keel will consist of "wing" devices.
However, some separation may be admitted for drag generation in downwind sailing, which can come out in the algebra as an increase in $C_{D_{0}}$.

Main conclusions are:

* It is possible to define nondimensional coefficients which describe the performance of some rig/keel combination and which depend mostly on only two variables.

* Conservative calculation of expected sailing-ship voyage times can be found by combining weather statistical data with rig/keel performance data.

* Tacking criteria for motor-sailers are given; the method of derivation may be used equally well to define other tacking criteria appropriate to some other consideration, such as minimum fuel usage or minimum operating cost.

* Expected fuel savings for a motor-sailer, with a fixed ship speed, can be found by combining weather statistical data with rig/keel/engine/hull data. Strategies other than fixed speed can be adopted to obtain greater savings using similar methods.
2. BACKGROUND CONSIDERATIONS

2.1 Wind Velocity Triangle – Basic Results

A wind velocity triangle is constructed, appropriate to wind velocity components approaching a ship.

Figure 1. Wind Velocity Triangle

\[ V_S, V_A \text{ and } \beta \text{ can be measured and the cosine rule may be applied to obtain:} \]

\[
\begin{align*}
V_T^2 &= V_S^2 + V_A^2 - 2 V_S V_A \cos \beta \\
V_T &= \sqrt{V_S^2 + V_A^2 - 2 V_S V_A \cos \beta} \quad (0 < \beta < \pi)
\end{align*}
\]

1.

The sine rule can be used to obtain:

\[
\frac{V_T}{\sin \beta} = \frac{V_A}{\sin \delta}
\]

2.

or \[ \gamma = \sin^{-1} \left( \frac{V_A \sin \beta}{V_T} \right) \]

Alternatively, \( V_A \) and \( \beta \) can be found from \( V_T \) and \( \gamma \) via

\[
V_A = \sqrt{V_S^2 + V_T^2 + 2 V_S V_T \cos \gamma}
\]

3.

and

\[ \beta = \sin^{-1} \left( \frac{V_T \sin \gamma}{V_A} \right) \]

4.
(3) and (4) may be combined to obtain:

\[
\sin \beta = \frac{V_t \sin \gamma}{\sqrt{V_s^2 + V_t^2 + 2V_s V_t \cos \delta}}
\]

(1) and (3) may be combined to obtain:

\[
\cos \beta = \frac{V_s + V_t \cos \gamma}{\sqrt{V_s^2 + V_t^2 + 2V_s V_t \cos \gamma}}
\]

In reality, wind speed is a function of height, so that \( \beta \) is a function of height, \( \beta = \beta(z) \). There are compensating corrections that can be applied, correct to the order of our approximations.

2.2 **Ship Resistance**

Ship upright water resistance may be obtained from:

\[
R_r = \frac{C \pi \rho S V^2 V_s}{250}
\]

In addition to (7), additional upright resistance may normally be expected from the action of wind and waves. Relative to \( R_r \), this is known to be small for light to moderate winds and calm seas. Consequently, this additional resistance will be ignored as far as the present analysis is concerned, implying a limitation on its validity in respect to ship speed predictions in heavy weather.

Moderate heeling, without sideforce, is known to have very little effect on ship resistance and will be ignored. However, heeling is known to increase the resistance due to sideforce and this effect could at some stage be taken into account. For larger ships, heeling is less a factor as with increasing linear dimension (e.g., length) the heeling moment increases by the third power of the length scale factor while the restoring moment increases by the fourth power.
Production of sideforce leads to a large increase in ship resistance due to induced drag. The nature of this increase varies, depending on how the ship sideforce ($F_s$) is produced. A ship hull is a low aspect lifting surface and produces compensating sideforce relatively inefficiently. Elaborate keels, commonplace on sailing vessels, are needed to generate lift in windward conditions. However, sail assisted vessels operating at or near a service speed will be able to generate significant sideforce by virtue of the speed of the fluid. Obviously practical considerations generally will encourage the avoidance of extra appendages. Experience on large sailing vessels of a half a century ago found that the addition of a skeg or a bar keel was often sufficient to reduce leeway to an acceptable amount.

An alternative is to use some form of keel, ideally having a smooth surface, deep draft and carefully designed planform. Analysis of such a keel may be achieved by means of an "equivalent keel" concept as described by Kerwin (1978).

Figure 2. Equivalent Keel Concept

To analyze the keel lift (or sideforce), standard methods may be applied to the equivalent lifting problem to obtain:

$$F_s = C_{lk} \frac{1}{2} \rho_s V_s^2 S_{ek}$$  

8.

$$AR_{ek} = \frac{2S^2}{S_{ek}}$$  

9.

$$R_i = \frac{k_k C_{lk}}{\pi AR_{ek}} \frac{1}{2} \rho_s V_s^2 S_{ek}$$  

10.
The parasitic drag of the keel can be accounted for by means of an equivalent keel parasitic drag coefficient $C_{D\text{OEK}}$. This consists of the keel parasitic drag coefficient ($C_{D\text{OEK}}$) scaled by the ratio of the immersed to equivalent keel areas as shown in Figure 3.

Figure 3. Immersed and Equivalent Keel Areas

![Diagram showing immersed and equivalent keel areas]

The parasitic resistance of a keel may be obtained as:

$$R_{\text{KE}} = C_{D\text{OEK}} \cdot \frac{1}{2} \rho \bar{V}^2 \alpha E_k$$

A few words about the implications of equations (8-11) are appropriate before continuing.

Some assessment of $F_S$ and $V_S$ is needed to ensure $C_{\text{LK}}$ has a reasonable value and the keel does not stall. The incidence or camber of the keel is conventionally fixed relative to the hull. In this case the keel area will be sized to keep hull leeway angles from causing significant drag increases due to separation of the hull flow.

However, it should be possible to engineer a rotatable keel whose incidence or camber can be varied relative to the hull to produce sideforce at zero ship leeway angle and allow reductions in the keel area and hence resistance. Such arrangements have been tried in the past and caused steering problems, but there do not appear to be insurmountable difficulties for the future. Our observations about the practicality of such devices still apply, however.

The implication of equation 11 is that deeper draft keels or faster ship speeds result in less induced resistance. One consequence of this is that when motorsailing to windward, the $F_S/R_i$ ratio of the hull is
improved, implying that pure sail experience sometimes inadequately represents the efficacy of motor-sail performance.

2.3 Marine Aerofoil Forces

The aerodynamic loads produced by a sail-rig can be extremely difficult to analyze, particularly if a lifting membrane (soft sail) is used as the major component. In the case of a wing-sail, circulation control or similar device, analysis is possible via lifting line or lifting surface methods. The essential aerodynamic problem is to model the marine aerofoil and hull above waterline together with an image system. In the case of a rig, where overall height is much greater than deck height, the reflection plane may be taken as the sea, giving an aerodynamic problem as shown in Figure 4.

Figure 4. Marine Aerofoil Aerodynamic Model

Analysis of this type of problem can be achieved by standard methods (Bergeson et al, 1981) with forces as described by:

\[ L = C_{lr} \frac{1}{2} \rho_a V_a^2 S e_r \]  
\[ D = C_{dr} \frac{1}{2} \rho_a V_a^2 S e_r \]  
\[ = \left[ C_{Dor} + k_r \frac{C_{lr}^2}{11 \ A_{re}} \right] \frac{1}{2} \rho_a V_a^2 S e_r \]

\[ C_{Dor} \] is an equivalent profile drag coefficient for the rig, calculated by taking \( C_{Dor} \) and scaling by the ratio of the "exposed" area of rig above the hull and the area of rig when projected down to the waterline.
From Figure 4,

$$ Ser = 5.6 $$

$$ \text{Aer} = 2.6^3 / \text{Ser} $$

and Kr is an induced drag factor dependent on rig span loading only.

Equation (3) may be used in conjunction with (12) and (14) to obtain rig forces:

$$ L = C_{Lr} \cdot \frac{1}{2} \rho_a \cdot (V_s^2 + V_r^2 + 2 V_s V_r \cos \beta) \cdot \text{Ser} $$

$$ D = \left[ C_{Dc} + Kr \cdot \frac{C_{Lr}^3}{11 \text{Aer}} \right] \cdot \frac{1}{2} \rho_a \cdot (V_s^2 + V_r^2 + 2 V_s V_r \cos \beta) \cdot \text{Ser} $$

Some of the implications of (17) and (18) need to be appreciated at this stage. For windward sailing, a reasonable L/D ratio is required, implying that span loading should be looked at closely. An elliptical loading maximises L/D; however, it may not be fully appropriate for a ship going to windward. This depends on the windward ability of the hull and many other factors. A good L/D ratio can also be achieved by increasing rig height which is reflected in (18) by an increase in ARer. This can also cause excessive heeling moments and stall the upper parts of a rig when a ship is rolling, resulting in a reduction in speed. The latter effect can be noticed with very small vessels in rough water but may not be too important for ships.

When reaching, the essential requirement is to maximize lift, and when running, to maximize drag. As with multiple keels, the basic forms of (17) and (18) can still be used to describe multiple aerofoils, and again some re-definition of Aer, Ser and Kr will be needed. When a vessel heels, there is a change in both the geometry of the force balance and the effective apparent wind angle \( \beta \), as shown in Figure 5.
Figure 5. Wind Velocity Triangle Changes Due to Heeling

This treatment of heeling is rather like that of an aircraft wing with dihedral, where an assumption is made that velocity components in the spanwise direction have no effect on the pressure distribution.

From Figure 5, the effective apparent wind angle is changed as a consequence of heeling and is now given by:

\[
\beta_e = \tan^{-1}\left\{\frac{\sin \beta \cos \phi}{\cos \beta}\right\}
\]

\[
= \tan^{-1}\left\{\tan \beta \cos \phi\right\}
\]

The importance of (19) is that it allows heeling to be removed from subsequent analysis since every problem involving it can now be related to an equivalent unheeled problem once \(\beta_e\) or \(\gamma_e\) is calculated.

2.4 Balance of Force on Sailing Ship

Rig and hull forces are in balance when a sailing ship moves through the sea/air at constant relative speeds. This equilibrium case is the only one to be considered and as a first step, attention is drawn to the problem of resolving rig lift and drag into a driving force \(F_R\) and heeling force \(F_H\). Figure 6 illustrates the rig forces.
Figure 6. Rig Forces

Forces may be resolved as:
\[ F_R = L \sin \beta - D \cos \beta \]
\[ F_H = L \cos \beta + D \sin \beta \]

Using (5) and (6) and nondimensional coefficients,

\[ F_R = \frac{1}{2} a_s S \cos \beta \sqrt{V_s^2 + V_r^2 + 2 V_s V_r \cos \beta} \cdot (C_L + V_r \sin \beta - C_w \sqrt{V_s + V_r \cos \beta}) \]
\[ F_H = \frac{1}{2} a_s S \cos \beta \sqrt{V_s^2 + V_r^2 + 2 V_s V_r \cos \beta} \cdot (C_L [V_s + V_r \cos \beta] + C_w V_r \sin \beta) \]

The hull forces are a sideforce \( (F_S) \) together with upright, keel parasitic and induced resistance \( (R_T, R_{OEK} \text{ & } R_i) \). This balance of forces is shown in Figure 7.

Figure 7. Balance of forces on sailing ship.
For the equilibrium case considered, clearly:

\[ F_R = R_T + R_{ex} + R_i \]  \hspace{1cm} 22.

\[ F_H = F_S \]  \hspace{1cm} 23.

If motor-sailing is being considered, a net thrust \( (T_R) \) from the propeller is included in \( (22) \) to obtain the general form:

\[ T_R + F_R = R_T + R_{ex} + R_i \]  \hspace{1cm} 24.
3. SIMPLIFICATION AND SOLUTION OF SAILING SHIP 
PERFORMANCE EQUATIONS

The objective of this section is to explain both the solution of sailing ship performance equations and concepts which seem useful for extensions to existing theory. Before proceeding, it is wise to review the assumptions made so far. These may be summarized:

* vertical wind velocity gradient effects ignored
* air drag on hull and wave resistance ignored
* hull resistance due to heeling ignored
* effects of heeling on rig forces can be accounted for separately
* residuary resistance coefficient is essentially constant in the speed range of sail assisted vessels

We recombine equations (20), (7) and (10) to rewrite the sailing vessel thrust/drag relationship.

\[
\frac{1}{2} \rho_s S_{er} \sqrt{V_s^2 + V_r^2 + 2 V_s V_r \cos \gamma} \cdot (C_{lr} V_r \sin \gamma - C_{or} [V_s + V_r \cos \gamma])
\]

\[
= \frac{C. \pi \rho_s V_s^3 \gamma}{250} + \left( C_{or} + k_k \cdot C_{ik} \right) \cdot \frac{1}{2} \rho_s V_s^2 S_{er} \frac{V_r}{V_t}
\]

Now if (25) is divided by \( 1/2 \rho_s V_T^2 S_{er} \) and a non-dimensional speed ratio parameter introduced as:

\[
\alpha = \frac{V_s}{V_T}
\]

then (25) becomes:

\[
\sqrt{\alpha^2 + 2 \alpha \cos \gamma} \cdot (C_{lr} \sin \gamma - C_{or} [\alpha \cos \gamma])
\]

\[
= \frac{C. \pi \rho_s a^3 \gamma}{250} + \left( C_{or} + k_k \cdot C_{ik} \right) \rho_s S_{er} \frac{a^2}{f_h}
\]

Hence the two variables \( V_s \) and \( V_T \) have now been replaced by a single variable - \( \alpha \). The implications will be discussed later. \( C_{ik} \) now needs to be related to \( F_H \). Using (8), (21), and (23):
\[ C_{\text{K}} = \frac{1}{2} \rho \, \text{S}_{\text{K}} \, \sqrt{V_s^2 + V_t^2 + 2 \, V_s \, V_t \, \cos \gamma} \left( C_{\text{Lr}} \left[ V_s + V_t \, \cos \gamma \right] + C_{\text{Dr}} \, V_t \, \sin \gamma \right) \]

\[ \frac{1}{2} \, \rho \, \text{S}_{\text{K}} \, V_s^2 \, \text{S}_{\text{K}} \]

\[ = \frac{\rho \, \text{S}_{\text{K}} \, \sqrt{a^2 + 1 + 2 \, a \, \cos \gamma} \left( C_{\text{Lr}} \left[ a + \cos \gamma \right] + C_{\text{Dr}} \, \sin \gamma \right)}{\text{S}_{\text{K}} \, a^2} \]

Substituting (28) into (27) gives:

\[ \frac{\sqrt{a^2 + 1 + 2 \, a \, \cos \gamma} \left( C_{\text{Lr}} \, \sin \gamma - C_{\text{Dr}} \left[ a + \cos \gamma \right] \right)}{\text{S}_{\text{K}} \, \pi \, a^2} \]

\[ = \frac{\openbox \, \text{S}_{\text{K}} \, \pi \, a^2 \, \sqrt{25}}{\rho \, \text{S}_{\text{K}} \, \text{S}_{\text{K}} \, \pi \, a^2} \]

\[ + \rho \, \text{S}_{\text{K}} \, \text{S}_{\text{K}} \, \pi \, a^2 \]

\[ + \frac{k \, \left( a^2 + 1 + 2 \, a \, \cos \gamma \right) \left( C_{\text{Lr}} \left[ a + \cos \gamma \right] + C_{\text{Dr}} \, \sin \gamma \right)^2}{\text{S}_{\text{K}} \, \pi \, a^2} \]

This equation is of similar form to that derived by Letcher (1982) in the nondimensionalization and the inclusion of keel parasitic drag term. Equation (29) is nonlinear in \( a \) and \( \gamma \). Other variables either depend on \( a \) and \( \gamma \) or are substantially constant. One exception is \( \bullet \). This will be substantially constant at low-ship speeds, and so for these speeds it is possible to conclude that the ratio of ship speed to true wind speed will also be constant for a given true wind angle, \( \gamma \).

In practice, (29) can be easily solved with a root-finding numerical technique, and a typical performance polar is shown in Figure 8.
Figure 8. Performance Polar for Fishing Boat
The motor-sailing case may be solved in a similar way, although in this instance a specific, rather than generalized answer will be obtained. One concept which is important for future work in motor-sailing is the idea of the net beneficial effect of the rig.

As before, a given rig/keel combination lends itself to nondimensional analysis and the following coefficients may be defined:

\[
\text{Cer} = \frac{F_R - R_k - R_m}{\frac{1}{2} \rho_a V^2 S_{er}} = \text{Cer}(a, \gamma)
\]

30.

\[
\text{Cep} = \frac{(F_R - R_k - R_m) V_S}{\frac{1}{2} \rho_a V^2 S_{er}} = \text{Cep}(a, \gamma)
\]

31.

The effective drive and power coefficients may be used either as yard sticks for the evaluation of rig/keel combinations or as a means of tabulating data applicable to superficially more complex problems.

Equation (29) may be decomposed and used with (30) and (31) to obtain:

\[
\text{Cer} = \sqrt{a^2 + 1 + 2a \cos \gamma} \left( C_{\text{Re}} \sin \gamma - \text{Cer} \left[ a + \cos \gamma \right] \right)
\]

\[
- C_{\text{weex}} \frac{f_S}{f_a} \frac{S_{ex}}{S_{er}} \frac{a^2}{S_{er}}
\]

\[
- k_x \frac{(a^2 + 1 + 2a \cos \gamma)}{1 + (C_{\text{Re}} \left[ a + \cos \gamma \right] + \text{Cer} \sin \gamma)^2} \frac{f_a}{f_a} \frac{S_{er}}{S_{er}} \frac{a^2}{S_{er}}
\]

32.

and

\[
\text{Cep} = a \cdot \text{Cer}
\]

33.

It is important to note that the values of \( \text{Cer} \) and \( \text{Cep} \) are independent of many of the simplifications and assumptions made to describe hull resistance; in particular the absence of \( \gamma \) is particularly gratifying as it is very difficult to assess.

Having said that it must be realized that this level of analysis has ignored problems such as: multiple mast aerodynamics, heavily loaded lifting surfaces, nonlinear hull induced drag effects and the windage effects on side force and hence induced drag.
Figure 9. Effective Power Polar - Fishing Boat

SYMBOL | a
-------|---
\(\circ\) | 0.16
\(\triangle\) | 0.47
\(\star\) | 0.78
\(\times\) | 1.10
Figure 10. Effective Power Polar - General Cargo Ship
In the motor-sailing mode, power can be varied to maximize either the fuel-saving or speed-enhancing benefits of the rig/keel.

The implications of treating the problem in this way should not go unnoticed. Effective drive and power coefficients can be readily used by naval architects to relate familiar upright ship resistance data to rig/keel data.

In practical use, \( C_e \) is defined for some rig/keel combination as a function of \( \alpha \) and \( \gamma \). To calculate sailing-ship speed, the equation:

\[
C_e (\alpha, \gamma) = \frac{\pi}{4} \cdot \frac{\rho \cdot \frac{1}{2} \cdot V_r^2 \cdot \sqrt{\frac{\phi}{250}}}{\frac{1}{2} \cdot \frac{1}{2} \cdot \rho \cdot \frac{1}{2} \cdot V_r^3 \cdot S_e} \]

must be solved. This is simply another form of (29) and may be solved in the same way. The form shown in (34) allows details of promising rig/keel combinations to be computed, stored on file and used as required with different hull forms. As mentioned above, knowledge of \( \phi \) for different hull forms as a function of speed is not often readily accessible.

The power coefficient may be used to assist with the calculation of fuel savings as follows:

Effective power from rig = \( \frac{C_e (\alpha, \gamma) \cdot \frac{1}{2} \cdot \rho \cdot \sqrt{\frac{\phi}{250}} \cdot V_r^3 \cdot S_e}{\eta_r \cdot \eta_D} \) 35.

Power required from engine to deliver the same effective power to hull = \( \frac{C_e (\alpha, \gamma) \cdot \frac{1}{2} \cdot \rho \cdot \sqrt{\frac{\phi}{250}} \cdot V_r^3 \cdot S_e}{\eta_r \cdot \eta_D} \) 36.

Fuel saved = \( \frac{S_F \cdot T_I \cdot C_e (\alpha, \gamma) \cdot \frac{1}{2} \cdot \rho \cdot \sqrt{\frac{\phi}{250}} \cdot V_r^3 \cdot S_e}{\eta_r \cdot \eta_D} \) 37.

This assumes that all effective power delivered can be profitably used; more will be made of this point later.
4. CALCULATION OF VOYAGE TIMES AND FUEL SAVINGS

The time a sailing vessel takes on a voyage depends essentially on winds, waves and currents it encounters and the actions of the crew. Fuel saved by a motor-sailer depends on the same parameters. Climatology can be used to obtain a probability density of having a wind speed and direction at a geographical location at a time of year. Knowing these probability densities, it then becomes possible to compute either an expected voyage time—under sail alone or an expected fuel saving using a known ship speed. The word "expected" is used in its mathematical sense: i.e., the sum of the products of the probabilities times the value in question.

From the ship owner's point of view, these calculations should indicate both the possible utilization of a pure sailing vessel and the potential economic benefits of a motor-sailer. Obviously, these figures relate to longer periods than individual voyages but should be quite adequate to decide whether the benefits of wind power are sufficient to justify further consideration.

The climatology we use for our calculations was obtained on computer tape from the National Climatic Center in Asheville, North Carolina and is the same information as plotted as wind roses on the Pilot Charts as depicted in Figure 11.

A probability density function (pdf) is defined such that:

\[ \int \int \rho f(v, \theta) d\theta dv = 1 \]

It needs also to be understood that:

\[ \rho f = \rho f(v_T, \theta: \text{geographical location, time of year}) \]

Equation (39) is combined with a ship performance polar to calculate an expected speed made good along some intended track. The form of this performance polar is split into two parts. Firstly, if (29) is solved for a constant \( \theta \), then a solution is obtained as \( a(\gamma) \). This does not account for tacking, and so a new "effective" solution to (29) is sought to account for this. Reference is now made to Figure 12.
Figure 11. Wind Roses on Pilot Chart (Northwest Atlantic)
Figure 12. Implications of Tacking

\[ v_t \]

\[ a_e = a_o \cdot \frac{X \cos(\gamma_0 - \theta) + Y \cos(\gamma_0 + \theta)}{X + Y} \]

since

\[ X \sin(\gamma_0 - \theta) = Y \sin(\gamma_0 + \theta) \]

\[ a_e = a_o \cdot \frac{\sin(\gamma_0 + \theta) \cos(\gamma_0 - \theta) + \cos(\gamma_0 - \theta) \sin(\gamma_0 + \theta)}{\sin(\gamma_0 - \theta) + \sin(\gamma_0 + \theta)} \]

\[ a_e = a_o \cdot \frac{\sin \beta \gamma_0}{2 \sin \gamma_0 \cos \gamma_0} \]

\[ a_e = a_o \cdot \frac{\cos \gamma_0}{\cos \gamma} \]

for \( |18| < |\gamma_0| \)

The solution to (29) may now be combined with (40) to produce an "effective" linearized speed ratio polar, \( a_{e_1}(\gamma) \), suitable for use with (38). See Figure 8.

Calculation of voyage time can now proceed by relating \( \gamma \), the true wind angle relative to track and the wind direction \( \theta \), relative to true north. If \( \theta_1 \) defines the required track, then:

\[ \gamma = \min \{ |\theta - \theta_1|, 2\pi - |\theta - \theta_1| \} \]
The expected ship speed \( (V_{ps}) \) may be found by combining (38), (41) and (44) to obtain:

\[
V_{ps} = \int_0^{\infty} \int_0^{2\pi} V_T \, q(\theta) \, \rho d_f(\theta, V_T) \, \vartheta \, d\theta \, dV_T
\]

Another way of approaching the tacking question which requires less mathematical manipulation and offers more insight into the issue is to recognize that any speed polar can be converted into an "effective" speed polar by choosing the maximum of i) the speed polar value itself at a given angle \( (\gamma) \) or ii) that value of a \( (or \, V_g) \) at the intersection of the radial line from the zero speed origin with a line drawn between any two points on the "ordinary" speed polar. Thus for windward sailing if we draw a line tangent to the speed polar at maximum speed points (port and starboard tacks), that line intersects the dead-to-windward case at the maximum speed made good point.

In auxiliary sail propulsion cases often occur where the minimum speed is not dead to windward but at some angle, say, broad off the bow. This would be brought about by the increased side force on the hull due large top hamper and freeboard, resulting in a significant increase in induced hull drag. This method of making the speed polar "convex" is easily adapted to a computer and guarantees the proper "effective" polar.

Conventional ship economics leads to the concept of an "economical ship speed" where the cost of burning extra fuel outweighs any economic advantage arising from increased utilization of the ship and her crew. It is possible to pursue this theme with a motor sailer, i.e. combine (37), (38) and (44), calculate time from an integral along the track of the reciprocal of "economic ship speed" to obtain:

\[
\text{expected fuel saving} = \int_0^{\infty} \int_0^{2\pi} \frac{\rho d_f(\theta, V_T) \cdot C_{ep}(V_T, \theta) \cdot \eta_f V_T^3 S_{er.} S_{FC} \cdot d\theta \, dV_T \, d\vartheta}{V_S \eta_\vartheta \eta_f}
\]

There are several drawbacks associated with (46). First and foremost is that it is based on a fixed-ship speed rather than average voyage time. If ship speed is allowed to vary, then greater advantage may be taken of the wind. Secondly, it over-estimates the fuel savings in extreme weather, and a limit should be placed on the effective power that can be delivered. Thirdly, implications behind the efficiency terms \( \eta_T \) and \( \eta_D \) require some comment. With a fixed-ship speed policy, the motor-sailer needs either a controllable-pitch or self-pitching propeller to maximize fuel saving, as power to be delivered by the engine is very variable. A multiple-engine
installation and possibly a gearbox may be used to guarantee the delivery of the required amount of effective power at a reasonable SPC. All of these refinements are present in some new ships and frequently justified by fuel savings they produce in their own right. For a motor-sailer, they are much more desirable and should also be less expensive as the installed engine power should be reduced. It might also be noted that a low-speed, direct-drive diesel coupled to a single fixed-pitch propeller may be used, but that the fixed ship speed policy would be very difficult to implement. Instead, a policy of operating the engine within a fixed economical power range and allowing the ship speed to vary around an acceptable mean, might be the best bet.

Clearly, the operation of a motor-sailer needs to be examined with much more care if the fuel-saving potential of the wind is to be properly utilized. Fuel-saving depends on the effective power delivered by the wind, time and the engine/transmission/propeller efficiency. Criteria are needed to decide whether or not to tack and if so, through what angle so as to develop more realistic "equivalent" \( C_{p} \) curves for all possible headings. Once these are obtained, expected fuel savings can be estimated on a sounder basis than the form shown in (46).

Firstly, however, reference is again made to Figure 12 with a view to finding a speed required during a tacking manoeuvre \( V_{S2} \) to equate to an equivalent speed made good \( V_{S1} \).

\[
\text{Time taken to steaming directly from A to C} = \frac{X \cos(Y - \gamma) + Y \cos(Y + \gamma)}{V_{S1}} \quad 47.
\]

\[
\text{Time taken to motor-sail route A-B-C} = \frac{X + Y}{V_{S2}} \quad 48.
\]

Equations (47) and (48) must clearly be equal if the tacking maneuver is to be a fair comparison with the direct route under power alone. Equating them gives rise to the expression:

\[
V_{S2} = V_{S1} \frac{\cos \gamma}{\cos \gamma_0} \quad 49.
\]

The tacking angle \( \gamma_0 \) depends essentially on what use the rig is being put to. If costs are to be minimized, then an objective function is set up which expresses cost and \( \gamma_0 \) varied until a minimum is found. For the present however, \( \gamma_0 \) will be found from minimum fuel usage.
The vessels used for examples in the computation of expected sailing speeds and fuel savings were 1.) a motor-sailing fishing vessel and 2.) a general cargo vessel. The fishing vessel is comparable to a design by Monk and built by Skookum Marine. It is 10.4 m over all with a displacement of approximately 17,600 lbs. For purposes of motor sailing we assumed it would be driven at 7 knots under combined sail and power. The general cargo vessel is similar to the SD4, a well known design of 14,000 tonnes deadweight and a speed of 14 kts. She was assumed to be fitted with a rig of 1,200 square meters.

The speed polar of Figure 8 shows fishing boat speed both with and without tacking. Since we have linearized the problem and have non-dimensionalized vessel speed \( V_s \) by true wind speed \( V_t \) to get \( a \), there is only one curve to be plotted. As discussed above, this representation is valid at low to moderate wind and ship speeds.

The power polars of Figures 9 and 10 for both the fishing boat and cargo ship depict the results of equation (33). Note that the origin represents a power loss of \( C_{ep} = -0.2 \), thus for high values of \( a \), both vessels show a net power loss (cost) attributable to the rig. Notice the large increase in net power delivered from beam winds.

The two vessels were "sailed" on typical routes for the purpose of observing the results of equations (45) and (46). The fishing boat was assumed to go from Tampa to 29N 92W to 22N 95W and return to Tampa (the "Gulf Route"). The cargo ship travelled a fictitious route from Tampa to Key West to the Northeast Providence Channel direct to Southampton and return via Bahamas - Key West.

The probability values, \( \text{pdf}(\theta, V_t) \), of the encountered wind speed and direction relative to the ship's bow were derived from the Pilot Chart wind probabilities, \( \text{pdf}(\theta) \), the Pilot Chart average wind speeds and application of equation (44). The distribution of wind speed for a given angle \( a(\gamma) \) (or \( V_t \)) was given by a two parameter Weibull function where the parameters are set by the mean wind speed and standard deviation of wind speed, both numbers from the National Climatic Center Pilot Chart data tape.

The Pilot Chart Climatological data are given uniquely for each 5 degree Marsden Square. As the vessel proceeds along its track, it traverses different Marsden Squares. A computer program was written that keeps track of the wind statistics along the ship's passage. Equations (45) and (46) were also programmed to give the expected passage speed and expected net fuel savings due to sail assist along the route.
Figure 13 is an example of computer output of that program for the fishing boat on the "Gulf Route". The ship speed was the expected ship speed given by Equation (45) assuming no auxiliary power was used. The fuel saved was determined by (46) assuming a fixed speed strategy of 7 knots. The program was run for each of the twelve months.

The general cargo ship was run on the "Southampton Route" for each of the twelve months. Equation (45) was not utilized as it would only be in exceptional cases that some sort of motor propulsion would not be used. The fuel savings in tonnes per voyage is tabulated in Table 1 along with the mean wind speed encountered along the voyage. Notice that as the wind speed varies so, of course, does the net fuel savings.

For both the fishing boat and the cargo ship it was assumed conservatively that the rigs would not be used in true wind speeds in excess of 36 knots. The choice of a rig design wind speed is economic (Bergeson et al). Since the probability of wind speed diminishes rapidly at higher wind speeds, there is little to be gained in designing a rig for power extraction at very high winds due to the greatly added capital cost of a stronger rig.
<table>
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<th>MONTH</th>
<th>AVERAGE WIND SPEED</th>
<th>NORTH</th>
<th>EAST</th>
<th>SOUTH</th>
<th>WEST</th>
<th>TOTAL DISTANCE</th>
<th>TOTAL ANGLES</th>
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**Note:** All tables and calculations are placeholders for demonstration purposes.
Table 1. Monthly Performance Statistics

<table>
<thead>
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<th>Month</th>
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<th>General Cargo Vessel (Southampton Route)</th>
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<tr>
<td></td>
<td>Boat Speed (kts)</td>
<td>Fuel Saved (tonnes)</td>
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<tr>
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</tr>
<tr>
<td>December</td>
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<td>.261</td>
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</table>

5. SUMMARY

We have presented a simplified approach for the analysis of sail assisted vessels. If we assume vessels operate the majority of the time in moderate wind conditions and at low Froude number, then sailing ship speed is essentially linear in wind speed for a given angle of wind off the bow. Thus a single speed polar $a = a(\gamma)$ or $ae = ae(\gamma)$, with tacking, can be used to describe sail-only performance.

Of perhaps greater interest, as it bears upon the economics, is the net thrust $C_{er}$ or net power $C_{ep}$ attributable to the existence of the sailing rig under different wind conditions. $C_{ep}$ embodies the added parasitic drag of the rig and added keel as well as thrust of the rig and the aerodynamically induced drag of the rig and hydrodynamically induced drag of the hull-keel as a lifting surface.

These equations were combined with Pilot Chart Climatology to see the fuel saving results over a hypothetical route in the Gulf of Mexico for a sail assisted fishing boat. Similar results were obtained for a general cargo ship with a sail assist rig on passages between Tampa and Southampton. The expected fuel savings were tabulated for each month showing the impact of time of year on the rig related fuel savings.
APPENDIX I

Calculations Involving Heeling

Heeling is defined as an angular displacement about an axis parallel to a ship's center line passing through its center of gravity. A body axis system is needed to resolve lift and drag into sideforce and longitudinal forces, see Figure AI.

Figure AI. Body Axis System for Heeling Calculations

Unit vectors \( y, \eta, s \) describe the apparent wind direction, lift direction and marine aerofoil orientation. These vectors are defined relative to \( x, y, z \) axes as:

\[
\begin{align*}
\mathbf{y} &= \begin{bmatrix} -\cos(\beta - \lambda) \\ \sin(\beta - \lambda) \\ 0 \end{bmatrix} \\
\mathbf{z} &= \begin{bmatrix} 0 \\ \sin \phi \\ \cos \phi \end{bmatrix} \\
\mathbf{\eta} &= \mathbf{y} \times \mathbf{z} = \begin{bmatrix} \sin(\beta - \lambda) \cos \phi \\ \cos(\beta - \lambda) \cos \phi \\ -\cos(\beta - \lambda) \sin \phi \end{bmatrix}
\end{align*}
\]

Force coefficients \( C_x \) and \( C_y \) relative to body axes are therefore:

\[
\begin{align*}
C_x &= C_L \sin(\beta - \lambda) \cos \phi - C_D \cos(\beta - \lambda) \\
C_y &= C_L \cos(\beta - \lambda) \cos \phi + C_D \sin(\beta - \lambda)
\end{align*}
\]

These expressions are useful for ship-motion and stability calculations. For performance calculations, coefficients \( C_R \) and \( C_H \) relative to the course sailed are found from:

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\[ C_x = C_x \cos \lambda + C_y \sin \lambda \]
\[ = C_x \cos \phi \left\{ \cos \lambda \sin (\beta - \lambda) + \sin \lambda \cos (\beta - \lambda) \right\} + C_y \left\{ \cos \lambda \cos (\beta - \lambda) + \sin \lambda \sin (\beta - \lambda) \right\} \]
\[ = C_x \cos \phi \sin \beta - C_y \cos \beta \]

\[ C_y = -C_x \sin \lambda + C_y \cos \lambda \]
\[ = C_x \cos \phi \left\{ -\sin \lambda \sin (\beta - \lambda) + \cos \lambda \cos (\beta - \lambda) \right\} + C_y \left\{ \sin \lambda \cos (\beta - \lambda) + \cos \lambda \sin (\beta - \lambda) \right\} \]
\[ = C_x \cos \phi \cos \beta + C_y \sin \beta \]

Heeling therefore erodes both drive and sideforce coefficients as a direct consequence of geometry, before any account is taken of its effect on \( C_L \) and \( C_D \). For some future wind powered ship design, it is probable that for windward sailing, where heeling is important, that \( C_L \gg C_D \) and \( \phi < 15^\circ \). Assuming \( C_L/C_D = 10 \), \( \beta = 30^\circ \) and \( \phi = 15^\circ \), then \( C_R \) will be reduced by \( 4\% \) and \( C_H \) by \( 3\% \). Since it appears likely that the aerodynamic effects of heeling can be related to changes in \( V_A \) and \( \beta \), similar corrections are sought for the geometric effects.

Writing:
\[ C_H = (1 - \frac{\Delta V}{V_A})^2 \left\{ C_x \cos (\beta - \delta) + C_y \sin (\beta - \delta) \right\} = C_x \cos \beta \cos \phi + C_y \sin \beta \]
\[ C_R = (1 - \frac{\Delta V}{V_A})^2 \left\{ C_x \sin (\beta - \delta) - C_y \cos (\beta - \delta) \right\} = C_x \sin \beta \cos \phi - C_y \cos \beta \]

allows the geometric effects of heeling to be interpreted in terms of a correction \( \delta \) to \( \beta \) and \( \Delta V \) to \( V_A \). The corrections are found from a root-finding technique and are small. For the windward case of \( C_L/C_D = 10 \), \( \beta = 30^\circ \), \( \phi = 15^\circ \), \( \delta_g = .22^\circ \), \( \Delta V_g = .017V_A \)

Aerodynamic effects of heeling are accounted for in a similar way using correction terms \( \delta_c \) and \( \Delta V_c \). Reference to Figure 5 shows:
\[ \delta_c = \beta - \tan^{-1} \left\{ \tan \beta \cos \phi \right\} \]
\[ \Delta V_c = V_A \left( 1 - \sqrt{\cos^2 \phi + \sin^2 \beta \cos^2 \phi} \right) \]

The heeled case can now be defined in terms of an equivalent unheeled problem having an equivalent apparent wind angle, \( (\beta_e) \) given by:
\[ \beta_e = \beta - \delta_g - \delta_c \]
and an equivalent apparent wind velocity \( (V_{ae}) \) given by:
\[
V_{ae} = V_a - AV_g - AV_c
\]

For many purposes, equivalent true wind angle \( (\gamma_e) \) and speed \( (V_{Te}) \) need to be used as input data. These may be found from:
\[
V_{Te} = \sqrt{V_s^2 + V_{ae}^2 - 2 V_s V_{ae} \cos \beta_e}
\]
\[
\gamma_e = \sin^{-1} \left( \frac{V_{ae} \sin \beta_e}{V_{Te}} \right)
\]

which are essentially equations (1) and (2) with the \( e \) suffix attached.

ACKNOWLEDGEMENT

The vector analysis at the start of this Appendix was contributed by Dr. J.F. Wellicome.

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SAFETY AND STABILITY CONSIDERATIONS
FOR SAIL ASSISTED FISHING VESSELS

by

William E. Remley
Commander, U.S. Coast Guard

ABSTRACT

The U.S. fishing fleet has suffered many accidents due to poor vessel stability. The magnitude and seriousness of these casualties are discussed. Since the U.S. Government has few rules for the safety of fishing vessels, some suggestions for better stability are set forth. For sail assisted fishing vessels the existing stability criteria used for regulated passenger and freight sailing vessels is presented. With the right considerations this method of assessing sail vessel stability could be applied to sailing or sail assisted fishing vessels. Some pertinent characteristics of sail vessels that meet the existing criteria are presented in tabular form. A sail assisted fishing vessel is checked against the sailing vessel stability criteria used for U.S. commercial vessels.

INTRODUCTION

Safety and stability of fishing vessels are very much entwined. Casualty data for the years 1972 through 1979 show an ever increasing number of vessels lost and deaths on fishing vessels attributed to foundering, flooding, and capsizing. In fact, the data shows that these types of accidents were the major source of casualties for vessels in this class. (2)

In the years 1978 and 1979, 144 people lost their lives in the three major casualty categories mentioned. Fires and explosions, groundings, collisions, and material failures contributed to the deaths of only 22 people over this same period. 169 vessels were lost to the first three categories while only 135 were lost in the last four. This translates to 85 people killed for every 100 vessels lost in a foundering, flooding, or capsizing accident, while only 16 died in fire and explosions, groundings, collisions, and material failures. (2) The latest statistics released for fiscal year 1980 indicate that the trend is continuing: of 60 people killed on fishing vessels, 44 were lost in foundering, capsizing, and flooding accidents. Clearly these types of casualties should be a focus of concern for those involved with improving fishing vessel safety. (5)

DISCUSSION

What is being done to reduce the high cost of these losses and the high death rate? The regulatory attitude in the U.S. has not changed for many years. Fishing vessels are classed as "uninspected commercial vessels". They are regulated under the Motorboat Act of 1940 (MBA-40) which is directed primarily at recreational motorboat safety. This Act limits regulatory authority to those few items specifically set forth in
the Act. This feature has made it difficult to adopt technological advances that have developed over the last 40 years to uninspected commercial vessels. The result of this law, as applied to commercial fishing vessels, has been to produce out-of-date and inadequate rules. Enforcement of the law is even more difficult since a vessel must be underway to be boarded and checked for compliance. (3)

In spite of the regulatory constraints, the Coast Guard and other Federal agencies have published a number of articles and circulars for use by the commercial fishing industry. Some of these are:


(2) Articles in the Proceedings discussing lessons gathered from the casualty data. In the February, 1982 edition, there was an article on stability problems associated with certain type of fishing vessels. In particular, problems with the East coast clam and scallop vessels and Alaska crabbing vessels were discussed. (4)

(3) The Coast Guard publishes circulars called Navigation and Vessel Inspection Circulars (NVIC's) which recommend safe practices for the marine industry. Those pertaining to commercial fishing vessels are:

(a) NVIC 3-76: Stability of Fishing Vessels, With IMO Recommendations for Intact Stability of Fishing Vessels.

(b) NVIC 4-82: Uninspected Commercial Vessel Safety.

(c) NVIC 17-82: Intact Stability of Small Vessels;

Recommendations.

Of these circulars, the first is probably most useful since it addresses most aspects of fishing vessel stability from initial GM to safe operating procedures to preserve stability.

Except for the problem of how to assess the stability of small vessels, i.e., those less than 79 feet (24.1 meters) long, there is enough literature available to assist the motor propelled fleet in ensuring sufficient stability. Additionally, even though the academic question of how to determine the safe stability of a small vessel is still unanswered, some of the practical recommendations available will apply to a great number of small vessels.

What stability recommendations are available to fishermen who want to rig their vessels with sail? There exists some sail vessel stability criteria that could be applied to sail assisted or sailing fishing vessels. The Coast Guard has used it for a number of years to evaluate the stability of commercial passenger and freight carrying sailing vessels. The development of this criteria was set forth in a paper presented in Washington, D.C. before the Chesapeake Section of the Society of Naval Architects and Marine Engineers on 2 March 1966. (1) The same criteria was published for comment in the Federal Register.
The Coast Guard has consolidated its various stability regulations into a new Subchapter S. The current regulations are scattered throughout various parts of Title 46 and 33 of the Code of Federal Regulations and are often difficult to locate as well as being redundant. Other rules such as sailing vessel stability appear for the first time as proposed rules for public comment. The sailing vessel and other stability rules were previously issued as Policy Statements and Interpretations. (8)

The proposed sailing vessel rules appear in Appendix A. For the person designing a sail assisted or just plain sailing fishing vessel, these criteria could be used to evaluate its stability. However, since the majority of fishing vessels are uninspected commercial vessels, it is not mandatory that they comply with these criteria.

Since the regulations only tell you how to do the calculations, a discussion of the criteria is helpful and might persuade some people to use them. The method used is a full range stability analysis evaluating three factors that affect sailing vessels: (1)

1. The first factor evaluated is initial stability of the vessel to sail comfortably and safely in normal weather. As shown in the Appendix, this factor is evaluated by the equation:

\[
\frac{1000 \ (W)(H2A)}{(A)(H)} = \text{FACTOR}
\]

2. The second factor evaluated is the ability of the vessel to resist gust and squall conditions without rolling to the point where water enters the hull. The equation used to evaluate this factor is:

\[
\frac{1000 \ (W)(H2B)}{(A)(H)} = \text{FACTOR}
\]

3. The third factor checks the ability of the vessel to survive in extremis: A knockdown or a capsize: This factor is checked by the following equation:

\[
\frac{1000 \ (W)(H2C)}{(A)(H)} = \text{FACTOR}
\]

The first factor was developed from a static or steady wind condition. The righting arm of the vessel over a range of heel is compared with a heeling arm due to wind acting over the same range. The righting arm is defined as \(GZ = f_1 \theta\) and heeling arm as \(HZ = f_2 \theta\). If the heeling arm is applied slowly, equilibrium is reached at \(\theta_1\) where \(f_1 \theta = f_2 \theta\). This is taken as the response of the vessel to a steady wind condition.

The second and third factors use an energy balance between heeling arm curve and righting arm curve. The vessel will roll until the energy added to the system by the heeling arm acting over a range of heel is equal to the energy absorbed by the rolling of the vessel. At \(\theta_2\) or when

\[
\int_{0}^{\theta_2} n f_1 \theta d\theta = \int_{0}^{\theta_2} n f_2 \theta d\theta
\]
these energies are equal and the vessel will reach maximum roll. Figure 1 illustrates this full range analysis.

There are a number of factors that affect the righting arm curve. Those that should be considered are listed below:

1. Use the worst case, usually the highest vertical center of gravity.

2. The curve is generally developed at light operating condition.

3. Low tankage for consumables or other items that would lower KG should be assumed empty or possibly 10% with slack surface.

4. A number of normal operating conditions should be evaluated to find the worst case.

5. Buoyancy credit can be given for a tight superstructure.

6. Careful note must be taken of heel angle where water can enter the hull through doors, hatches, ports, and companionways. If the deck arrangement is assymetrical, the worst case is considered.

7. Trim with heel must be evaluated. This must be accounted for when correcting the righting arm curve.

8. The curve should be developed to at least 90 degrees. If the vessel has a range of stability in excess of 90 degrees, the curve is developed to the limit of the positive range or 120 degrees, whichever is less.

9. When developing the righting arm curve for fishing vessels, their unique operating conditions should be considered. If the catch is carried on deck, this case must be considered. Other conditions such as suspended fish loads, ice accumulation, shifting cargo, and effect of free liquids all must be considered. It is impossible to predict what the worst case for each type of vessel will be without careful evaluation. NVIC 3-76 and NVIC 17-82 offer guidance for developing some fishing vessel operating conditions.

When considering the effect of the wind, it is a beam wind acting on the entire area of the vessel above water. All sails are considered trimmed flat. As the vessel heels, both the projected area exposed to the wind and the arm at which the wind force acts decrease as the cosine of the heel angle. This establishes the equations for the heeling arm curve as

\[ H = H(A,B,C) \cos^2(T) \]

where \( H(A,B,C) \) is the heeling arm at zero degrees of heel for the various factors. \( H \) is the heeling arm and \( T \) is the angle of heel.

The evaluation of the three factors are based on heel angles of the vessel. The first factor for steady heel uses the heel angle at the deck edge, since most sailors will heel a vessel to deck edge under normal sailing conditions. For the second factor, the downflood angle or 60 degrees is used. This is the point where water could enter the hull if the vessel is hit unexpectedly with strong gusts of a squall. The 60 degrees limit is taken as a maximum since at this angle cabin gear and
BASIC PRINCIPLES OF FULL RANGE STABILITY ANALYSIS

FIG. 1
other inside ballast starts to come loose. The third factor or knock-
down goes to 90 degrees but never more than 120 degrees. If the range
of positive stability is less than 90 degrees, the area under the right-
ing arm curve is considered as a "negative area" in obtaining the balance.
If the range of positive stability exceeds 90 degrees, the additional area
up to the limit of positive stability, but never over 120 degrees, is in-
cluded. The heeling arm is considered as zero past 90 degrees. The full
knockdown case assumes the vessel can withstand a knockdown without down-
flooding. This assumption is based on the belief that a full knockdown
often occurs when the vessel is secured for heavy weather with all deck
openings closed. (1)

After the three cases are checked, the resulting numbers must equal
or exceed the stability worth numerals shown in Appendix A. For fishing
vessels, the numbers for vessels on exposed waters would be the most
appropriate.

Although other methods of evaluating sailing vessel stability exist,
they depend on initial response to assumed wind pressure. (1) It is felt
the full range analysis set forth in the proposed rulemaking is a simple,
straight forward, and comprehensive method for checking sail assisted or
sail powered fishing vessel stability.

However, a note of caution when using this criteria. It was origi-
nally developed for sailing vessels in the commercial passenger trade and
for vessels with cargo securely stowed on deck or within the holds. A
fishing vessel with a shifting cargo on deck (fish catch) or suspended
loads (nets suspended from outriggers) or towing nets will require that
these conditions be considered along with the sail criteria. NVIC 3-76
and NVIC 17-82 provide some guidance for considering these effects.

APPLICATIONS

The criteria for sailing vessels as it appears in the proposed rules
has been used to evaluate many commercial sailing vessels subject to Coast
Guard regulation. A summary of their characteristics is shown in Figure 2.
All are monohull vessels that were given stability tests to determine
lightship data, i.e., location of vertical and longitudinal center of
gravity and lightship displacement.

A 38 foot (11.6 meters) fishing vessel of recent design was given a
stability check to see if it satisfied the existing Coast Guard sail vessel
stability criteria. A side profile and particulars of the vessel are shown
in Figure 3. Since the vessel was not given a stability test, assumed
values for the vertical center of gravity were used. The vessel was eval-
uated at a full load displacement of 14 long tons (14.2 metric tons) and
draft of 2.89 feet (.88 meters) measured from the baseline.

The first case evaluated the vessel as designed with a deck edge
immersion angle of 8.0 degrees and downflood angle of 15.4 degrees. The
point of downflood was taken as the door sills on the side of the pilot
house. The vertical centers of gravity (VCG's) were taken from 2.0 to
4.0 feet (.61 to 1.22 meters) above the baseline.
# FIGURE 2

**COMMERCIAL SAILING VESSEL DATA**

<table>
<thead>
<tr>
<th>VESSEL</th>
<th>LENGTH ON THE WATER LINE FEET (METERS)</th>
<th>MAX. BEAM FEET (METERS)</th>
<th>DEPTH FEET (METERS)</th>
<th>SAIL AREA $^2$ FEET$^2$ (METERS$^2$)</th>
<th>LIGHTSHIP (INCL. BALLAST) LONG TONS (METRIC TONS)</th>
<th>FIXED BALLAST WEIGHT LONG TONS (METRIC TONS)</th>
<th>% OF LIGHTSHIP</th>
<th>TYPE RIG</th>
<th>SERVICE/ROUTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65.0 (19.8)</td>
<td>21.0 (6.4)</td>
<td>9.3 (2.83)</td>
<td>3020 (280.56)</td>
<td>75.0 (76.2)</td>
<td>20.0 (20.32)</td>
<td>26.7</td>
<td>SCOOOLER</td>
<td>PASSENGER/ OCEANS</td>
</tr>
<tr>
<td>2</td>
<td>53.3 (16.25)</td>
<td>18.75 (5.72)</td>
<td>13.6 (4.15)</td>
<td>2114.0 (196.39)</td>
<td>62.0 (63)</td>
<td>17.9 (18.19)</td>
<td>28.9</td>
<td>SCOOOLER</td>
<td>PASSENGER/ OCEANS</td>
</tr>
<tr>
<td>3</td>
<td>52.0 (15.85)</td>
<td>15.25 (4.65)</td>
<td>10.2 (3.11)</td>
<td>1914.0* (177.81)*</td>
<td>29.45 (29.92)</td>
<td>8.73 (8.87)</td>
<td>30.0</td>
<td>SCOOOLER</td>
<td>CARGO AND PASSENGER/ PARTIALLY PROTECTED**</td>
</tr>
<tr>
<td>4</td>
<td>55.0 (16.76)</td>
<td>18.67 (5.69)</td>
<td>10.48 (3.19)</td>
<td>1632.0 (151.61)</td>
<td>35.4 (35.97)</td>
<td>8.04 (8.17)</td>
<td>22.6</td>
<td>SLOOP***</td>
<td>PASSENGER/ OCEANS</td>
</tr>
<tr>
<td>5</td>
<td>60.0 (18.29)</td>
<td>21.0 (6.4)</td>
<td>8.83 (2.69)</td>
<td>2277.0 (211.53)</td>
<td>50.85 (51.66)</td>
<td>22.32 (22.68)</td>
<td>43.9</td>
<td>SCOOOLER</td>
<td>PASSENGER/ OCEANS</td>
</tr>
</tbody>
</table>

* WITH CARGO HOLD EMPTY TO 4.5 LONG TONS (4.57 METRIC TONS), 1556.0 SQUARE FEET (142.69 SQUARE METERS) OF SAIL MAY BE CARRIED.

WITH 4.5 LONG TONS (4.57 METRIC TONS) TO FULL LOAD, 1914.0 SQUARE FEET (177.81 SQUARE METERS) OF SAIL MAY BE CARRIED.

** CARGO CAPACITY IS 13½ LONG TONS (13.72 METRIC TONS).

*** FOR OCEAN SERVICE SAILS MUST BE REEFED.
38' SAIL ASSIST

38 FOOT (11.6 METER) SAIL ASSISTED FISHING VESSEL

FIG. 3

LENGTH OVERALL (LOA) 37.5 FT (11.43) M
LENGTH WATER LINE (LWL) 33.3 FT (10.15) M
BEAM 13.0 FT (3.96) M
DRAFT 5.0 FT (1.52) M
FISH HOLD 6,500 POUNDS (2,948.4) KG
SAIL AREA 734 FT. SQ. (68.19) M²
LIGHTSHIP DISPLACEMENT 22,500 POUNDS (10,206.0) KG
BALLAST (LEAD)* 7,000 POUNDS (3,175.2) KG

*8 OF LIGHTSHIP 31%
The second case evaluated assumed a modification to the vessel. The doors were moved as far inboard as possible so that the downflooding angle was 22.2 degrees.

For case 3, the downflood angle was increased to the maximum allowed or 60 degrees. The doors would have to be removed to achieve this and entrance to the pilot house provided through a hatch.

Table 1 shows the results of the three cases investigated. VCG is in feet (meters) above the baseline and the factors are in tons/square foot (metric tons/square meter).

<table>
<thead>
<tr>
<th>VCG FT (M)</th>
<th>FACTOR 1</th>
<th>FACTOR 2</th>
<th>FACTOR 3</th>
<th>FACTOR 2</th>
<th>FACTOR 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0 (.61)</td>
<td>.587 (6.42)</td>
<td>.528 (5.78)</td>
<td>3.47 (37.96)</td>
<td>.733 (8.02)</td>
<td>1.765 (19.31)</td>
</tr>
<tr>
<td>2.5 (.76)</td>
<td>.525 (5.74)</td>
<td>.470 (5.14)</td>
<td>2.632 (28.79)</td>
<td>.646 (7.07)</td>
<td>1.468 (16.06)</td>
</tr>
<tr>
<td>3.0 (.91)</td>
<td>.462 (5.05)</td>
<td>.411 (4.50)</td>
<td>1.793 (19.62)</td>
<td>.558 (6.10)</td>
<td>1.171 (12.81)</td>
</tr>
<tr>
<td>3.5 (1.07)</td>
<td>.400 (4.38)</td>
<td>.353 (3.86)</td>
<td>1.064 (11.64)</td>
<td>.471 (5.15)</td>
<td>.875 (9.57)</td>
</tr>
<tr>
<td>4.0 (1.22)</td>
<td>.338 (3.70)</td>
<td>.294 (3.22)</td>
<td>.491 (5.37)</td>
<td>.380 (4.16)</td>
<td>.650 (7.11)</td>
</tr>
</tbody>
</table>

**ANALYSIS OF SAIL ASSISTED FISHING VESSEL**

**TABLE 1**

For ocean service the resulting numerals must be equal to or exceed:

<table>
<thead>
<tr>
<th></th>
<th>English Units</th>
<th>Metric Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor 1</td>
<td>-</td>
<td>1.5</td>
</tr>
<tr>
<td>Factor 2</td>
<td>-</td>
<td>1.7</td>
</tr>
<tr>
<td>Factor 3</td>
<td>-</td>
<td>1.9</td>
</tr>
</tbody>
</table>

The results show that this vessel has excellent ability to survive in extremis. With some modifications, it's ability to avoid downflooding could be improved. Factor 1 could be improved through the modification of the freeing ports or by raising the main deck. If the freeing ports were modified to act as one way ports, i.e., only let water out and not in, the deck edge immersion angle would be increased. The ability of the existing design to reef her sails is also a great asset since decreasing the sail area increases the resulting stability numerals.

This vessel has been in service for quite some time and has not suffered any casualties. She has operated in severe weather and seen substantial beam winds with her sails set and trimmed flat. From all reports she withstood this test without any harm. She was not designed to meet the
Coast Guard proposed sail vessel criteria although with some changes it seems the existing design could be modified to meet all the criteria. With her ability to roller reef her sails, it seems that one of the most viable alternatives would be to correlate the amount of sail carried with wind velocity. Operational instructions for the crew could advise them when to reef their sails and how much sail they could safely carry.

In conclusion, it should be pointed out that the sailing vessel stability criteria discussed was developed to check commercial passenger and freight vessels using sails. Before attempting to apply these criteria to sail or sail assisted fishing vessels, the stability problems unique to fishing vessels must be incorporated. For example, a vessel with a load of fish on deck that could shift poses a unique problem. The shifting problem would have to be solved or a safety factor incorporated into the criteria to allow for the shifting cargo.

Anyone interested in evaluating the stability of fishing vessels should obtain a copy of the paper on the sail vessel stability criteria and copies of the Navigation and Vessel Inspection Circulars 17-82, 3-76, and 4-82. The paper "On the Stability of Sailing Vessels" can be obtained from the Society of Naval Architects and Marine Engineers, One World Trade Center, Suite 1369, New York, New York, 10048. The NVIC's can be obtained by writing to the address shown for them in the references.

The opinions expressed in this paper are those of the author and do not necessarily reflect those of the United States Coast Guard.
ACKNOWLEDGEMENTS

I would like to thank the following people for their cooperation which made this paper possible.

LTJG Randy GILBERT, U.S.C.G., Staff Engineer for the Third Coast Guard District Merchant Marine Technical Division, located on Governors Island, New York, New York. Randy supplied much of the data on sailing vessels he had evaluated for Coast Guard approval and did the stability evaluation of the 38 foot sail assisted fishing vessel.

Rob LADD, Naval Architect, and Chief Designer with Skye Marine Corporation, Fort Lauderdale, Florida. Rob supplied the specifications and plans of the 38 foot sail assisted fishing vessel as well as factual accounts of how the vessel was performing.

REFERENCES


APPENDIX A

PROPOSED RULES FOR SAIL OR SAIL ASSISTED VESSELS FROM FEDERAL REGISTER
OF
THURSDAY, AUGUST 12, 1982 - VOL. 47, NO. 156

Federal Register / Vol. 47, No. 156 / Thursday, August 12, 1982 / Proposed Rules 35101

Subpart E—Weather Criteria

§ 170.150 Specific applicability.
This subpart applies to each vessel except—
(a) A passenger vessel, other than a vessel the stability of which is questioned by the OCMH that—
1 Is less than 100 gross tons;
2 Is less than 19.8 meters LOD measured over the weather deck; and
3 Carries 49 or less passengers.
(b) A tank vessel, other than a vessel the stability of which is questioned by the OCMH that carries a product listed in § 30.25-1 of this chapter and that is—
1 Less than 150 gross tons; or
2 A tank barge that operates only in river or lakes, bays, sounds service.
(c) A tank barge that carries a product listed in Table 151.01-10(b) of this chapter.
(d) A mobile offshore drilling unit.
(e) A vessel that performs the tests required by § 171.050(c) of this chapter.
(f) A barge that complies with the requirements in § 174.020 of this subchapter.

§ 170.170 Calculations required.
(a) Each vessel must be shown by design calculations to have a metacentric height (GM) that is equal to or greater than the following in each condition of loading and operation:

\[
\text{GM} \geq \text{PAH} \tan (\theta)
\]

where—
\[
P = 0.055 \cdot (L/1200)^3 \text{ metric tons/m}^2 \ldots \text{for ocean service, Great Lakes winter service, or service on exposed waters.}
\]
\[
P = 0.050 \cdot (L/1200)^3 \text{ metric tons/m}^2 \ldots \text{for Great Lakes summer service or service on partially protected waters.}
\]
\[
P = 0.025 \cdot (L/1200)^3 \text{ metric tons/m}^2 \ldots \text{for service on protected waters.}
\]

\[\text{L} = \text{LBP in meters.}
\]

\[\text{A} = \text{projected lateral area in square meters of the portion of the vessel above the waterline.}
\]

\[\text{H} = \text{the vertical distance in meters from the center of A to the center of the underwater lateral area or approximately to the one-half draft point.}
\]

\[W = \text{displacement in metric tons.}
\]

\[T = 14 \text{ degrees or the angle of heel at which one-half the freeboard to the deck edge is immersed, whichever is less.}
\]

(b) If approved by the Commander (mmt), a larger value of T may be used for a vessel with a discontinuous weather deck of abnormal shear.

(c) When doing the calculations required by paragraph (a) of this section for a sailing vessel or auxiliary sailing vessel, the vessel must be assumed—
1 To be under bare poles; or
2 If the vessel has no auxiliary propulsion, to have storm sails set and trimmed flat.

(d) The criteria specified in this section are generally limited in application to flush deck, mechanically powered vessels of ordinary proportions and form that carry cargo below the main deck. On other types of vessels, the Commander (mmt) requires calculations in addition to those in paragraph (a) of this section. On a vessel under 100 meters in length, other than a tugboat or a towboat, the requirements in § 170.173 are applied. Additional intact stability requirements for tugboats and towboats are included in Part 174 of this subchapter.

§ 170.173 Criteria for vessels of unusual proportion and form.
(a) If approved by the Commander (mmt), each vessel less than 100 meters LLL, other than a tugboat or towboat, must be shown by design calculations to comply with—
1 Paragraph (b) or (c) of this section if the maximum righting arm occurs at an angle of heel less than or equal to 30 degrees; or
2 Paragraph (b) of this section if the maximum righting arm occurs at an angle of heel greater than 30 degrees.
(b) Each vessel must have—
1 An initial metacentric height (GM) of at least 0.15 meters;
2 A maximum righting arm (CZ) of at least 0.20 meters at an angle of heel equal to or greater than 30 degrees;
3 A maximum righting arm that occurs at an angle of heel not less than 25 degrees;
4 An area under each righting arm curve of at least 3.15 meter-degrees up to an angle of heel of 30 degrees; and
5 An area under each righting arm curve of at least 3.15 meter-degrees up to an angle of heel of 40 degrees or the downflooding angle, whichever is less; and
6 An area under each righting arm curve between the angles of 30 degrees and 40 degrees, or between 60 degrees and the downflooding angle if this angle is less than 40 degrees, of not less than 1.72 meter-degrees.
(c) Each vessel must have—
(1) An initial metacentric height (GM) of at least 0.15 meters;
(2) A maximum righting arm that occurs at an angle of heel not less than 15 degrees;
3 An area under each righting arm curve of at least 3.15 meter-degrees up to an angle of heel of 40 degrees or the downflooding angle, whichever is less;
4 An area under each righting arm curve between the angles of 30 degrees and 40 degrees, or between 60 degrees and the downflooding angle if this angle is less than 40 degrees, of not less than 1.72 meter-degrees; and
5 An area under each righting arm curve up to the angle of maximum righting arm of not less than the area determined by the following equation:

\[A = K1 + K2 (X - Y)\]

where—
\[A = \text{area in meter-degrees.}
\]

\[K1 = 3.15 \text{ meter-degrees.}
\]

\[K2 = 0.001 \text{ meter-degrees/degree.}
\]

\[X = 30 \text{ degrees.}
\]

\[Y = \text{angle of maximum righting arm, degrees.}
\]

(d) For the purpose of demonstrating compliance with paragraphs (b) and (c) of this section, at each angle of heel a vessel's righting arm is calculated after the vessel is permitted to trim free until the trimming moment is zero.
Subpart B—Small Vessels

§ 171.020 Specific applicability.

(a) Except as provided in paragraph (b) of this section, this subpart applies to each vessel that is less than 100 gross tons, less than 19.8 meters (65 feet) measured over the weather deck, and carries 150 or less passengers.

(b) This subpart does not apply to a vessel described in paragraph (a) of this section that carries more than 12 passengers on an international voyage.

§ 171.030 Intact stability requirements for a mechanically propelled or a nonself-propelled vessel.

(a) This section applies to each vessel, except a sailing vessel or an auxiliary sailing vessel, that—

(1) Carries more than 49 passengers;

(2) The stability of which is questioned by the OCCM; or

(3) Is permitted an increased passenger allowance by § 170.01-25(b) of this chapter.

(b) Each vessel must—

(1) Comply with § 171.050 and § 170.01-25 of this subchapter; or

(2) Perform the test in paragraph (d) of this section in the presence of the OCCM.

(c) Each vessel must be in the following condition when the test in paragraph (d) is performed:

(1) The construction of the vessel must be complete in all respects.

(2) Ballast, if necessary, must be solid and must be on board and in place.

(3) Fuel and water tanks must be approximately three-quarters full.

(4) The weight of passengers and other loads must be on board and distributed so as to provide normal operating power and to simulate the vertical center of gravity causing the least stable condition that is likely to occur in service. The number of passengers used in determining the total passenger weight must not be more than the maximum number permitted by § 170.01-25 of this chapter.

(d) If a vessel has non-return closures on cockpit scuppers or on weather deck drains, the closures must be kept open during the test.

(2) Each vessel must not exceed the limitations in paragraph (e) of this section, when subjected to the greater of the following heeling moments:

\[ M_p = \frac{W_p}{P} \]

or

\[ M_w = P A \]

where—

\[ M_p = \text{Passenger heeling moment in kilograms-meters,} \]

\[ W = \text{the total passenger weight in kilograms.} \]

\[ \text{Assume 33.8 kg per passenger on protected waters when passenger load consists of men, women, and children. Assume 3.4 kg per passenger all other times.} \]

\[ P = \text{Wind heeling moment in kilograms-meters.} \]

\[ A = \text{Area, in square meters, of the projected lateral surface of the vessel above the waterline (this surface includes each projected area of the hull, superstructure, and area bounded by sailings and structural canopies).} \]

\[ H = \text{Height, in meters, to the center of area, above the waterline.} \]

(3) Each vessel must not exceed the following limits of heel when doing the test in paragraph (d) of this section:

(1) On a flush deck or well deck vessel, the freeboard must be measured to the top of the weatherdeck at the side of the vessel.

(2) For a vessel with a cockpit or for an open boat, the freeboard must be measured to the top of the gunwale.

(b) On protected or partially protected waters—

\[ i = \frac{1}{2}(L - 15.27) \]

where—

\[ i = \text{maximum allowable immersion in meters.} \]

\[ L = \text{LOD, measured over the weather deck, in meters.} \]

\[ L = \text{length of cockpit in meters.} \]

(8) On an open boat, no more than one-quarter of the freeboard may be immersed.

(4) In no case may the angle of heel exceed 14 degrees.

(5) The limits of heel must be measured at—

(1) The point of minimum freeboard,

(2) At a point three-quarters of the vessel's length from the bow if the point of minimum freeboard is aft of this point.

(8) Each ferry must also be tested in a manner acceptable to the OCCM to determine whether the trim or heel during loading or unloading will submerge the deck edge. A ferry passes this test if the deck edge is not submerged during loading or unloading.

(9) When demonstrating compliance with paragraph (e) of this section, the freeboard must be measured as follows:

(1) For a flush deck or well deck vessel, the freeboard must be measured to the top of the weatherdeck at the side of the vessel.

(2) For a vessel with a cockpit or for an open boat, the freeboard must be measured to the top of the gunwale.
§ 171.035 Intact stability requirements for a sailing vessel or an auxiliary sailing vessel.

(a) Except as provided in paragraph (b) of this section, each of the following sailing vessels and auxiliary sailing vessels must meet the intact stability standards of § 171.055 and § 170.170 of this subchapter:

(1) A vessel to be operated in exposed waters.

(2) A vessel to be operated during non-daylight hours.

(3) A vessel of unusual type or rig.

(4) A vessel that carries more than 49 passengers.

(b) A catamaran must meet the intact stability requirements of § 171.057 and § 170.170 of this subchapter.

(c) Each sailing vessel and auxiliary sailing vessel not listed in paragraph (a) or (b) of this section must comply with the requirements in paragraphs (d) through (f).

(d) Each vessel must remain afloat when flooded or capsized.

(e) Each vessel must have suitable hand holds or other means to allow a person to cling to the vessel in the event of a capsizing.

(f) Each vessel operating in partially protected waters must have a self-bailing cockpit.

(g) The OCMIE determines whether the vessel has adequate stability for protected waters or partially protected waters. When making this determination, the analysis techniques of paragraphs (h) or (i) are used unless the OCMIE determines that other analysis techniques are more appropriate.

(b) Operational tests may be performed to assure that the vessel shows satisfactory handling characteristics under sail.

(i) The simplified stability test of § 171.030 may be used. The heel moment used for this test must be the greater of the following:

(1) Passenger heel moment from § 171.030.

(2) Wind heel moment from § 171.030 under bare poles, or, if the vessel has no auxiliary power, with storm sails set.

(3) Wind heel moment calculated from the following equation:

\[ M_w = \frac{PAH}{H} \]

where:

\[ M_w \] = wind heel moment in kilogram-meters.

\[ A \] = the windage area of the vessel in square meters with all sail set and trimmed flat.

\[ H \] = the distance in meters from the center of the windage area to the waterline.

\[ P = 4.9 \] for both protected and partially protected waters.

(j) Additional or different stability requirements may be needed for a broad, shallow draft vessel with little or no ballast outside the hull. The additional requirements, if needed, will be prescribed by the appropriate Commander (nni).
Subpart C—Large Vessels
§ 171.045 Specific applicability.
This subpart applies to each vessel that falls into any one of the following categories:
(a) Greater than 100 gross tons.
(b) Greater than 19.8 meters in length.
(c) Carries more than 12 passengers on an international voyage.
(d) Carries more than 150 passengers.
(e) The stability of which is questioned by the OCML.

§ 171.050 Intact stability requirements for a mechanically propelled or nonself-propelled vessel.
Each vessel must be shown by design calculations to have a metacentric height (GM) in meters in each condition of loading and operation, that is not less than the value given by the following equation:
\[
GM = \frac{N_{b}}{23.5 W \tan(T)}
\]
where:
N = number of passengers.
W = displacement of the vessel in metric tons.
T = 14 degrees or the angle of heel at which the deck edge is first submerged, whichever is less.
b = distance in meters from the centerline of the vessel to the geometric center of the passenger deck on one side of the centerline.

§ 171.055 Intact stability requirements for a monohull sailing vessel or a monohull auxiliary sailing vessel.
(a) Except as specified in paragraph (b) of this section, each monohull sailing vessel and auxillary sailing vessel must be shown by design calculations to meet the stability requirements of this section.
(b) Additional or different stability requirements may be needed for a vessel of unusual form, proportion, or rig. The additional requirements, if needed, will be prescribed by the Commandant.
(c) Each vessel must have positive righting arms in each condition of loading and operation from—
(1) 0 to at least 70 degrees of heel for service on protected or partially protected waters; and
(2) 0 to at least 90 degrees of heel for service on exposed waters.
(d) Each vessel must be designed to satisfy the following equations:
(1) For a vessel in service on protected or partially protected waters—
\[
1000W \times HZA = \frac{(A[H])}{(A[H])}
\]
>12.0 (metric tons/sq. meter)
\[
1000W \times HZB = \frac{(A[H])}{(A[H])}
\]
>13.7 (metric tons/sq. meter)
(2) For a vessel on exposed waters—
\[
1000W \times HZA = \frac{(A[H])}{(A[H])}
\]
>18.1 (metric tons/sq. meter)
\[
1000W \times HZB = \frac{(A[H])}{(A[H])}
\]
>18.0 (metric tons/sq. meter)
\[
1000W \times HZC = \frac{(A[H])}{(A[H])}
\]
>30.8 (metric tons/sq. meter)
where—
HZA, HZB and HZC are calculated in the manner specified in paragraph (e) or (f) of this section in meters.
A = the projected lateral area in square meters of the portion of the vessel above the waterline computed with all sail set and brimmed flat, except that 100% of the foretriangle area may be used in lieu of the area of the individual headsails when determining A if the total area of the headsails exceeds the foretriangle area.
H = the vertical distance in meters from the center of A to the center of the underwater lateral area or approximately to the one-half draft point.
W = the displacement of the vessel in metric tons.
(e) Except as provided in paragraph (f) of this section, HZA, HZB, and HZC must be determined as follows for each condition of loading and operation:
(1) Plot the righting arm curve on Graphs 171.055(b1)(c), (d) or (e).
(2) If the angle at which the maximum righting arm occurs is less than 35 degrees, the righting arm curve must be truncated as shown on Graph 171.055(a).
(3) Plot an assumed righting arm curve on Graph 171.055(b2) that satisfies the following:
(i) The assumed righting arm curve must be defined by the equation—
\[
H = HZA \cos^{T}(T)
\]
where—
H = heeling arm.
HZA = heeling arm at 0 degrees of heel.
T = angle of heel.
(ii) The first intercept shown on Graph 171.055(b2) must occur at the angle of heel corresponding to the angle at which deck edge immersion first occurs.
(4) Plot an assumed heeling arm curve on Graph 171.055(c) that satisfies the following conditions:
(i) The assumed heeling arm curve must be defined by the equation—
\[
H = HZA \cos^{T}(T)
\]
where—
HZA = heeling arm at 0 degrees of heel.
H = heeling arm.
T = angle of heel.
(ii) The area under the assumed heeling arm curve between 0 degrees and the downflooding angle or 90 degrees, whichever is less, must be equal to the area under the righting arm curve between the same limiting angles.
(5) Plot an assumed heeling arm curve on Graph 171.055(d) or (e) that satisfies the following conditions:
(i) The assumed heeling arm curve must be defined by—
\[
H = HZA \cos^{T}(T)
\]
where—
HZA = heeling arm.
HZA = heeling arm at 0 degrees of heel.
T = angle of heel.
(ii) The area under the assumed heeling arm curve between the angles of 0 and 90 degrees must be equal to the area under the righting arm curve between 0 degrees and—
(A) 90 degrees if the righting arm are positive to an angle less than or equal to 90 degrees; or
(B) the largest angle corresponding to a positive righting arm but no more than 120 degrees if the righting arms are positive to an angle greater than 90 degrees.
(6) The values of HZA, HZB, and HZC are read directly from Graphs 171.055(b1), (c), and (d) or (e).
(f) For the purpose of this section, the downflooding angle means the static angle from the intersection of the vessel’s centerline and waterline in calm water to the first opening that cannot be rapidly closed watertight.
(g) HZB and, if the righting arms are positive to an angle of 90 degrees or greater, HZC may be computed from the following equation:
\[
HZB \text{ or } HZC = \frac{I}{(1/2) + 14.3 \sin(2T)}
\]
where—
I = the area under the righting arm curve to—
(1) the downflooding angle or 60 degrees, whichever is less, when computing HZB; or
(2) 90 degrees or more but no more than 120 degrees when computing HZC.
The downflooding angle or 60 degrees, whichever is less, when computing HZB or 90 degrees when computing HZC.
GRAPH 171.055(a)

Truncation of Righting Arm Curve if Maximum Righting Arm Occurs at an Angle of Heel Less Than 25 Degrees
First Intercept Occurs at the Angle at Which Deck Edge Immersion First Occurs

\[ HZ = HZ\cos^2(\theta) \]

Righting Arm Curve

Shaded Areas are Balanced to the Downflooding Angle

\[ HZ = HZ\cos^2(\theta) \]

Righting Arm Curve
§ 171.057 Intact stability requirements for a sailing catamaran.

(a) A sailing catamaran that operates on protected and partially protected waters must be designed to satisfy the following:

\[
\frac{0.8(W)}{B} \geq \frac{2(As)(Hm)}{\text{m}^2} > 10.0 \text{ metric tons/m}^2.
\]

where—

\( B \) = the distance between hull centerlines in meters.
\( As \) = sail area in square meters.
\( Hm \) = the mast height above the deck in meters.
\( W \) = the combined displacement of both hulls in metric tons.

(b) A sailing catamaran that operates on exposed waters must be designed to satisfy the following equation:

\[
\frac{0.8(W)}{B} \geq \frac{2(As)(Hm)}{\text{m}^2} > 16.4 \text{ metric tons/m}^2.
\]

where—

\( B \) = the distance between hull centerlines in meters.
\( As \) = sail area in square meters.
\( Hm \) = the mast height above the deck in meters.
\( W \) = the combined displacement of both hulls in metric tons.

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BIOGRAPHY