A LABORATORY STUDY OF
THE FLUID-STRUCTURE INTERACTION
OF SUBMERGED TANKS AND
CAISSONS IN EARTHQUAKES

by
ROBERT C. BYRD

Report to Sponsors:
NOAA, Office of Sea Grant
The State Resources Agency of California

COLLEGE OF ENGINEERING
UNIVERSITY OF CALIFORNIA - Berkeley, California
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Report to

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ABSTRACT

An experimental study comparing the results of measurements of forces on a submerged tank model due to earthquake excitation is presented. The experimental results are compared with analytical solutions for the case where the model is submerged in water of depth equal to 2.5 times the tank height and for the case where the depth exactly equals the height.

Details are presented for the design of a 1 to 100 scale model of a circular cylindrical structure which is 34 meters in height with a mass of approximately 250,000 tons. The model includes a foundation system which simulates elastic half-space soil stiffness in three degrees of freedom.

The experimental results are presented in the form of inertia coefficients measured in harmonic motion at varying amplitudes and over a frequency range of 0.3 Hz to 2 Hz in prototype scale. Coefficients are presented for horizontal, vertical, rotational, and horizontal-rotational coupling. The relationship between these coefficients and the physics of the fluid-structure interaction are discussed in detail.

The study leads to the following conclusions concerning earthquake induced forces on large submerged, gravity-type structures:

a. Available analytical techniques provide good estimates of hydrodynamic inertia force coefficients for submerged structures of simple form.

b. A correct estimate of foundation damping is likely to be the most critical point in calculating the hydrodynamic forces on a submerged gravity structure.
c. Foundation stiffness only influences the hydrodynamic force by changing the resonant frequency.

d. Frequency dependence in the inertia coefficients is not likely to be an important consideration.

e. Coupling in the hydrodynamic inertia forces between the horizontal and rotational modes is not likely to be an important consideration in structural design.

f. Hydrodynamic dampening will not be an important factor for deeply submerged structures but may be significant in near surface and surface-piercing structures.
ACKNOWLEDGEMENT

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Professor C. J. Garrison of the Naval Postgraduate School, Monterey, California, was kind enough to perform the diffraction theory calculations for comparison with these experiments. This work has been a major contribution to the scope of this study.

The author is indebted to the faculty investigators for this research project, Professors Ben C. Gerwick, Jr., Joseph Penzien, and Robert L. Wiegel, for providing the necessary advice and guidance during the progress of this research and to Professor William C. Webster for many helpful suggestions in preparing the experiment and performing the analysis. Professor Jorg Imberger helped significantly in developing the proper data analysis techniques.

Special thanks are extended to Professor Wiegel who served as dissertation committee chairman and advisor throughout the author's progress through the doctoral program.

It would not have been possible to conduct this investigation without the able assistance of Mr. Fukij Nilrat in designing the model and carrying out the experiments and without the excellent facilities and personnel of the Earthquake Engineering Research Center.
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LIST OF SYMBOLS AND DEFINITIONS

[A] = structure projected area matrix

\( a \) = amplitude of ground motion
= \( u_g \), etc.

\( a_o \) = dimensionless frequency parameter for soil stiffness evaluation
= \( \omega R / c_s \)

[C] = foundation dampening matrix

[C_B] = hydrodynamic inertia coefficient matrix
= \( \rho K w A \)

C_{ii} = foundation dampening in mode i, neglecting coupling
= \( C_{ii}^B \)

C_{ij} = foundation dampening in mode i due to motion in mode j

C_{ij}^M = inertia coefficient for force in mode i due to motion in mode j

C_{ii}^r = hydrodynamic dampening in mode i, neglecting coupling

C_{ii}^* = hydrodynamic dampening in mode i associated with relative motion between structure and foundation

C_{ii0}^* = hydrodynamic dampening in mode i associated with rigid motion of the structure with the foundation

C_s = shear wave velocity in the soil

\( \tilde{C} \) = total dampening for a given mode, neglecting coupling
= \( C_{ii} + C_{ii}^* \)

D = structure diameter

F_{ii} = total force in mode i

F_{ij} = force in mode i due to motion in mode j

F_{Di} = hydrodynamic force in mode i due to drag

F_{II} = hydrodynamic force in mode i due to inertia effects
F_{wi} = hydrodynamic force in mode i due to wave generation

F_{b1} = complex frequency dependent force on the structure base in mode i

f_{b1} = time dependent force on the structure base in mode i

G = soil shear modulus of elasticity

g = acceleration of gravity \(-9.807 \text{ m/sec}^2\)

H = structure height

H' = effective height of the pressure distribution due to wave generation

h = water depth

\Delta H = depth of submergence = h - H

[K] = foundation stiffness matrix

[K_d] = hydrodynamic drag coefficient matrix

[K_m] = [C^m] = hydrodynamic inertia coefficient matrix

[K_w] = wave making coefficient matrix

\lambda_i = foundation stiffness in mode i neglecting coupling

\lambda_{b1} = \lambda_i

\lambda_{ij} = foundation stiffness in mode i due to motion in mode j

k = wave number

= \frac{2\pi}{\lambda}

L = foundation spring length

[M] = structure mass matrix

[M^*] = "virtual" or "added" mass matrix due to surrounding fluid

= \rho[C^m][V]

M_i = structure mass in mode i

M_{1i} = virtual mass in mode i associated with relative motion between the structure and foundation

M_{10} = virtual mass in mode i associated with rigid motion of structure with the foundation

M_{ij} = virtual mass in mode i for motion in mode j
\( \ddot{M} \) - total mass for a given mode, neglecting coupling
\[ = M_I + M^*_I \]

\( P_{wi} \) - pressure due to wave generation by structure motion in mode \( i \)

\( P^* \) - time dependent hydrodynamic pressure force

\( p^* \) - complex frequency dependent hydrodynamic pressure force

\( P^*_I, P^*_R \) - imaginary and real parts, respectively of the complex hydrodynamic force

\( p_i \) - time dependent hydrodynamic pressure force due to motion in mode \( i \)

\( p_i \) - total complex frequency dependent hydrodynamic pressure due to motion in mode \( i \)

\( p_{iI} \) - complex hydrodynamic pressure due to relative motion in mode \( i \) between the structure and foundation

\( p_{iI} \) - complex hydrodynamic pressure due to rigid motion of the structure and foundation in mode \( i \)

\( R \) - structure radius

\( R_g \) - radius of gyration of the displaced volume of water around its center of gravity
\[ = \left( \frac{b}{4} \right)^{\frac{2}{3}} \left( \frac{b^3}{12} \right) \text{ for a circular cylinder} \]

\( \{ r \} \) - time dependent displacement vector, relative to the foundation

\( \{ r_t \} \) - time dependent total displacement

\( S_{ij} \) - foundation spring stiffness in mode \( i \) for displacement in mode \( j \)

\( t \) - time

\( \{ u_q \} \) - time dependent foundation displacement vector, relative to a fixed reference

\( \{ u_w \} \) - time dependent water partial displacement, relative to a fixed reference

\( u_q \) - horizontal component of foundation displacement

\( [v] \) - volume matrix

\( v_q \) - vertical component of foundation displacement
$X_s$ = horizontal location of the foundation springs from the center of gravity

$\bar{X}$ = complex frequency dependent horizontal displacement amplitude, relative to the foundation

$x$ = time dependent horizontal displacement, relative to the foundation

$z$ = time dependent vertical displacement, relative to the foundation

$Z_{cg}$ = vertical location of structure center of gravity

$Z_s$ = vertical location of foundation spring attachment point from the center of gravity

$\rho$ = mass density of water

$\rho_s$ = mass density of soil

$\xi_i$ = foundation dampening in mode $i$, percent of critical

$\xi^*_i$ = hydrodynamic dampening in mode $i$, percent of critical

$\theta$ = angular displacement relative to the foundation

$\theta_g$ = angular component of foundation displacement

$\phi$ = linear velocity potential or phase angle, depending on context

$\sigma$ = dimensionless frequency parameter

$$\sigma = \frac{\omega^2 - \rho H}{g}$$

$\lambda$ = length of generated waves

$\omega$ = radial frequency

$= 2\pi/T$, where $T$ is the period in seconds
1. INTRODUCTION

The progress of offshore development on the North American west coast, in Alaska and around the Pacific Ocean in general in recent years has made it increasingly likely that large volume, gravity-type structures will be desirable in the near future in some applications in areas of high seismic activity. Since this type structure has had little or no prior history in this environment, it was considered desirable to conduct a series of experiments in as realistic conditions as possible in a laboratory to confirm or deny currently used analytic procedures for calculating earthquake forces. The overall purpose of this study is to reduce the uncertainty associated with the fluid-structure interaction aspect of these calculations by answering some of the questions concerning the inertia coefficients.¹

1.1 Review of Analytical Procedures

The details of the analytical procedures will not be discussed in this paper, but it is appropriate to give some consideration to the techniques which are generally used in engineering applications and to the types of problems which they solve. These procedures can be lumped broadly into three categories.

a. closed form (or continuum) solutions,

b. diffraction theory based on the use of Green's functions,

c. variational methods (finite element).

Diffraction and variation methods can be considered as closed-form solutions under some circumstances², but in application they

* Superscripts refer to the corresponding items under 'References'.

involve discretization of the system and are essentially numerical procedures. Wehausen and Laitone\cite{3} provide a detailed discussion of the basis for all of these procedures, examples of the application of diffraction theory are shown by Garrison and Chow\cite{6} and by Hogben and Standing\cite{5}, Bai\cite{6} and Zienkiewicz and Newton\cite{7} show examples of fluid problem solutions applying the variational principle to finite elements. Liaw and Chopra\cite{8} also present a variational method solution for the fluid problem along with a closed-form solution for comparison. Petrauskas\cite{9} presents a similar closed-form solution.

The assumptions that are generally common to these methods are:

a. small amplitude displacements such that linear boundary conditions may be assumed,

b. inviscid fluid (irrotational flow),

c. incompressible fluid (except as shown in Refs. 7 and 8.

However, the effects of compressibility can be shown to be negligible for the type of structure and motion presently being considered).

Solutions to the fluid problem under these circumstances can be considered as "linear potential flow" solutions, and it can be shown by comparison of the results under similar conditions that the solutions by any of these methods are comparable, as one would hope. All methods are not, however, available under all circumstances with closed-form solutions being limited to simple geometries and diffraction and closed-form solutions being limited to harmonic motions, linear superposition not withstanding.

It has been shown that all of these methods yield good results under conditions in waves which satisfy their assumptions (see Refs. 6, 9 and 10, for examples). The experiments presented in this report
have attempted to test these assumptions under realistic earthquake conditions. Diffraction calculations were performed by Garrison specifically for comparison with these experiments in the fully submerged condition and a closed-form solution after Liaw was performed for the condition where the structure penetrated the surface.

1.2 Objectives and Scope of the Investigation

The study has concerned itself with examining the following factors:

a. The general degree to which analytic procedures can accurately predict hydrodynamic earthquake forces.

b. The existence of significant frequency dependence over the range of frequencies of interest for earthquakes.

c. The influence of coupling between modes within the hydrodynamic inertia forces.

d. The sensitivity of the hydrodynamic forces to changes in the various coefficients.

It was desirable to stay as general and simple as possible in selecting a prototype system to model so that the results would be interpretable and broadly applicable--while retaining enough realism to make the effort worthwhile. The following system was selected to satisfy these ends:

a. A circular cylindrical tank, or gravity structure caisson without a superstructure, of approximately 250,000 tons with a height approximately equal to its radius.

b. An elastic but firm foundation.

c. A water depth of approximately 100 meters.

The idealized prototype system is shown in Fig. 1.1.
(a) PROTOTYPE SYSTEM

(b) MODEL IDEALIZATION

FIGURE 1.1: PROTOTYPE SYSTEM AND MODEL IDEALIZATION FOR THE SUBMERGED TANK EXPERIMENT
2. DYNAMICS OF OFFSHORE GRAVITY STRUCTURES IN EARTHQUAKES

2.1 The Equation of Motion

It is appropriate to consider briefly the general equation of motion for a structure in the marine environment, in this case generally following the matrix notation of Penzien.\(^{11}\)

\[
\begin{align*}
[M] \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{r}_t \end{bmatrix} + [C] \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{r}_t \end{bmatrix} + [K] \begin{bmatrix} r \\ \end{bmatrix} &= \\
\rho [K_m-1] [V] \begin{bmatrix} \ddot{u}_w-\dot{x} \\ \ddot{u}_w-\dot{y} \\ \ddot{u}_w-\dot{z} \\ \ddot{u}_w-\dot{r}_t \end{bmatrix} + \rho [V] \begin{bmatrix} \dot{u}_w \\ \end{bmatrix} \\
&+ \rho [K_w] [A] \begin{bmatrix} \dot{u}_w-\dot{x} \\ \dot{u}_w-\dot{y} \\ \dot{u}_w-\dot{z} \\ \dot{u}_w-\dot{r}_t \end{bmatrix} + \rho [K_d] [A] \begin{bmatrix} (\ddot{u}_w-\dot{x}) \\ (\ddot{u}_w-\dot{y}) \\ (\ddot{u}_w-\dot{z}) \\ (\ddot{u}_w-\dot{r}_t) \end{bmatrix}.
\end{align*}
\tag{2.1}
\]

In the general case for an offshore gravity structure, we would be concerned about flexibility in the platform and legs, but we can usually treat the base as being rigid. Since we are only considering the base response in this study we will only concern ourselves with the rigid body response modes. Therefore, we can define:

\[
\begin{align*}
[A] &= \text{structure projected area matrix}, \\
[C] &= \text{foundation damping matrix}, \\
[K] &= \text{foundation stiffness matrix}, \\
[K_d] &= \text{hydrodynamic drag coefficient matrix}, \\
[K_m-1] &= [c^m] = \text{hydrodynamic inertia coefficient matrix}, \\
[K_w] &= \text{wave making coefficient matrix}, \\
[M] &= \text{structure mass matrix}, \\
\{r\} &= \text{displacement vector, relative to the moving foundation}, \\
\{r_t\} &= \text{total displacement} = \{r\} + \{u_g\},
\end{align*}
\]
\{u_g\} = foundation displacement vector, relative to a fixed reference,
\{u_w\} = instantaneous water particle displacement relative to a fixed reference,
\[v\] = volume matrix.

The experimental work to be discussed was conducted under conditions of initially still water, and we may, therefore, drop water particle motions which are normally caused by waves from further consideration.

2.2 Scaling of Forces

At this point, one should consider the relative magnitudes of the various forces acting on the structure in an earthquake, as defined by Eq. 2.1. We must, therefore, further define the conditions under which we will seek a solution.

Physical considerations indicate that there are six parameters (excluding viscosity) which influence the earthquake forces on marine structures. These are (see Fig. 1.1):

\(a = \) amplitude of ground motion = \(u_g\), etc.,
\(\omega = \) radial frequency of the motion = \(2\pi/T\), where \(T\) is the period in seconds,
\(h = \) water depth,
\(D = \) diameter of the structure = \(2R\),
\(H = \) height of the structure,
\(g = \) acceleration of gravity = 9.807 m/sec\(^2\).
2.2.1 Forces in Horizontal Motion

Examination of the right-hand side of Eq. 2.1 shows that there are three types of forces: (a) inertial, (b) drag, and (c) wave making. If we assume harmonic motion, we can describe the magnitude of these forces with the following approximate relationships for the horizontal mode of motion:

\[ F_x = F_{Ix} + F_{Dx} + F_{Wx} \]

and,

\[ F_{Ix} = \rho (K_{wx} - 1) V \tilde{u} = \rho \frac{\pi D^2}{4} H_a \omega^2 \]

\[ F_{Dx} = \rho K_{Dx} A \tilde{u} \dot{u} = \rho D H_a^2 \omega^2 \]

\[ F_{Wx} = \rho K_{wx} A \tilde{u} \dot{u} = DH^\prime p_{wx} \]

where: \( \rho \) = mass density of the water,

\( F_W \) = pressure on the structure due to waves being generated by the structure motion,

\( H^\prime \) = effective height of the pressure distribution on the structure.

MacCamy and Fuchs² have shown that there are two components of pressure due to a structure oscillating horizontally in a fluid: (a) one associated with the inertia force term and due to local disturbance of the fluid by the structure, (b) a second in phase with the velocity and due to creation of progressive waves of the same frequency as the oscillations and which transmit energy from the system. The progressive wave pressure term has been shown to extend to an effective depth approximately equal to the wave length, \( \lambda \).

If we assume a wave amplitude approximately equal to the amplitude of the ground motion, we can describe the velocity potential for the
linear progressive wave as follows:  

\[
\phi = \frac{aw}{k} \cosh (kz) \sin (kx - \omega t) \ldots \ldots \ldots \ldots \ldots
\]

where:

\[
k = \frac{2\pi}{\lambda} = \frac{\omega^2}{g} \tanh (kh) \ldots \ldots \ldots (2.5)
\]

The amplitude of the wave making pressure term then becomes:

\[
P_{Wx} = \rho \frac{\partial \phi}{\partial t} = \frac{\rho a^2 \omega^2}{k} \ldots \ldots \ldots (2.6)
\]

We can assume that for all practical cases involving gravity structures and earthquakes,

\[
kh >> 2\pi.
\]

Therefore,

\[
k = \frac{2\pi}{\lambda} = \frac{\omega^2}{g} \ldots \ldots \ldots (2.7)
\]

and

\[
\lambda = \frac{2\pi a}{\omega^2} \ldots \ldots \ldots (2.8)
\]

We may now rewrite Eq. 2.4 as follows:

\[
P_{Wx} = \frac{\rho a^2 \omega^2}{k} \Delta H = \rho a g \Delta H \ldots \ldots \ldots (2.9)
\]

Defining,

\[
\Delta H = h - H = \text{depth of submergence}.
\]

Then, Eq. 2.9 becomes,

\[
P_{Wx} = \rho agD(\lambda - \Delta H), \ldots \ldots \ldots (2.10)
\]

where the wave force amplitude is recognized to be positive only.

We are now in position to consider the relative magnitude of the various forces for horizontal motion. Dividing the sum of the forces
by the inertia term we now have:

\[
F_{x//F_{ix}} = \frac{4\rho D H \omega^2}{\rho n D^2 H a} \omega^2 + \frac{1 + 4\rho a g c(\lambda - \Delta H)}{\rho n D^2 H a \omega^2} \quad \ldots \ldots (2.11)
\]

It is immediately clear that the drag term will not be important, since we are generally considering amplitudes of ground motion much less than one meter and structure diameters of approximately 100 meters. It would appear that structure member diameter would have to be in the order of one or two meters before drag would need to be considered in earthquake motion. This agrees with the general range of importance for drag in waves as found by other investigations.\(^5\)

Consideration of the wave making term shows a somewhat more complex situation. We can, however, see immediately from Eq. 2.10 that energy will not be dissipated by wave making for any structure where the depth of submersion exceeds the wave length, or:

\[
\Delta H < \frac{2\pi q}{\omega^2} \quad \ldots \ldots (2.12)
\]

For the case where structure height equals water depth (\(\Delta H = 0\)), the wave making force ratio in Eq. 2.11 becomes:

\[
\frac{F_{wX}}{F_{ix}} = \frac{g\lambda}{D H \omega^2} = \frac{2\pi g^2}{D H \omega^b} = \frac{2\pi}{\sigma^2} \quad \ldots \ldots (2.13)
\]

where,

\[
\sigma = \frac{\omega^2 q D H}{g} \quad \ldots \ldots (2.14)
\]
We can conclude that where the dimensionless frequency parameter, \( \sigma \), is large, little energy will be dissipated through wave making, regardless of the depth of submergence, \( \Delta h \). However, a considerable amount of energy could be dissipated by wave making at low frequencies and shallow submergence depths.

2.2.2 Forces in Vertical Motion

The forces due to vertical motion of the structure can be written approximately as:

\[
P_z = P_{iz} + P_{dz} + P_w
\]

and,

\[
P_{iz} = \rho (K_m - 1) \nabla \dot{\nabla} = \rho \frac{\pi D^2}{4} H_a \omega^2 \quad \ldots \ldots \quad (2.15)
\]

\[
P_{dz} = \rho K_{dz} A \ddot{\nabla} = \rho \frac{\pi D^2}{4} a^2 \omega^2 \quad \ldots \ldots \quad (2.16)
\]

\[
P_w = \rho K_{wz} A \ddot{\nabla} = \frac{\pi D^2}{4} \frac{\rho}{\omega} \quad \ldots \ldots \quad (2.17)
\]

We note that the wave pressure force again is intended to describe the effect of waves propagating from the system. Unfortunately, we do not have a convenient expression to describe this quantity. However, we can define the region in which it is likely to become important by noting that the wave length of the highest frequency wave which could propagate completely from the vicinity of the disturbance caused by structure motion would be equal to the diameter of the structure, i.e.

\[
\lambda_{critical} = D \quad \ldots \ldots \quad (2.18)
\]

This follows from the relationships between wave length, propagation speed, and frequency in deep water.\(^{12}\) We can now define a critical
frequency above which we would expect energy loss due to wave making
to decrease rapidly:

\[ \omega_{\text{critical}} = \left( \frac{2\pi f}{D} \right)^{1/2} \] \hspace{1cm} (2.19)

Examination of the ratio of drag to inertia force yields

\[ \frac{\frac{\partial D}{\partial r}}{\frac{\partial I}{\partial r}} = \frac{a}{H} \] \hspace{1cm} (2.20)

Once again we can conclude that the drag force will be negligible for
most gravity structures.

2.3 The Virtual Mass Representation of Fluid Effects

We can now rewrite the equation of motion in simplified form as:

\[ [M] \ddot{x} + [C] \dot{x} + [K] x = -[M^*] \ddot{x} + [C^*] \dot{x}, \] \hspace{1cm} (2.21)

with notation as before except:

\[ [M^*] = \text{matrix of the "virtual" or "added" mass of the surrounding water} \]

\[ = \rho [c^N] [v] \]

\[ [C^*] = \text{coefficient of equivalent hydrodynamic damping, due primarily to wave making at shallow water depths} \]

\[ = \rho [k_w] [A] \]

The purpose of this investigation was primarily to shed additional light on the virtual mass matrix \([M^*]\) under earthquake conditions and to compare the findings with analytical methods for computing these effects.

The term "virtual mass" is used in this report in the same context as "added mass". Virtual mass is favored as a term for describing the inertial effect of the surrounding fluid because, depending on the usage, the effect is not always additive.
It is now convenient to dispense with the matrix notation and write Eq. 2.21 as three independent mode equations:

\[ M_x \dddot{x} + C_x \ddot{x} + K_x x = -M^*_x \dddot{x}_t - M^*_x \dddot{q}_t - C^*_x \dddot{x}_t \ldots \ldots (2.22) \]

\[ M_z \dddot{z} + C_z \ddot{z} + K_z z = -M^*_z \dddot{z}_t - C^*_z \dddot{z}_t \ldots \ldots (2.23) \]

\[ M_\theta \dddot{\theta} + C_\theta \ddot{\theta} + K_\theta \theta = -M^*_\theta \dddot{\theta}_t - C^*_\theta \dddot{\theta}_t \ldots \ldots (2.24) \]

It would be expected that the coupled virtual mass terms \( M^*_{x0} \) and \( M^*_0 \) are approximately equal and that they are not particularly large. This last hypothesis will be supported by the test results which we shall discuss later. The rotational acceleration, \( \dddot{x}_t \), is also very small in the experimental system and we would expect that the coupled force term would drop out of Eq. 2.22.

2.3.1 Hydrodynamic Pressure

Before we proceed with the discussion of the submerged tank experiment, it is enlightening to consider the fundamentals of the virtual mass representation of fluid effects on structures. The following discussion will be limited to the rigid body mode of horizontal motion on a flexible foundation; interested readers are referred to Chapter 2 of Liaw and Chopra for a discussion of the effects of structure flexibility modes on virtual mass.

The uncoupled equation for horizontal motion including fluid effects can be written (see Fig. 1.1):

\[ M_x \dddot{x}(t) + C_x \ddot{x}(t) + K_x x(t) = -M^*_x \dddot{x}_t(t) - P^*_x(t) \ldots \ldots (2.25) \]

where \( P^*_x(t) \) represents the force in the horizontal mode of oscillation associated with the hydrodynamic pressure, \( p_x(z, \phi, t) \). This quantity is described by the Laplace equation in cylindrical coordinates,
\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \phi^2} + \frac{\partial^2 p}{\partial z^2} = 0 \quad \ldots \ldots \ldots (2.26)
\]

which, after applying appropriate boundary conditions, can be solved for the dynamic pressure distribution on the surface of simply shaped structures which pierce the surface. If such a solution is available, then

\[
P_x(t) = \int \int \int_{0}^{\pi} (z, \phi, t) R \cos \phi \, d\phi \, dz \quad \ldots \ldots \ldots (2.27)
\]

2.3.2 The Complex Frequency Response Representation

One characteristic of linear systems with time independent physical properties is that they respond to simple harmonic excitation with simple harmonic motion of the same frequency, once steady-state conditions have been achieved. The amplitude and phase relationship of this response is frequency dependent in general. This frequency dependence is conveniently described by the use of complex frequency response functions. The complex response function \( x(t) \) is written as \( \tilde{x}(\omega) \) and has the property that when the excitation is the real part of \( e^{i\omega t} \), the response is the real part of \( \tilde{x}(\omega) e^{i\omega t} \).

Applying this to the hydrodynamic pressure from Eq. 2.27, we have for the pressure on the surface of an oscillating structure

\[
P_x(z, \phi, t) = \tilde{P}_x(z, \phi, \omega) e^{i\omega t} \quad \ldots \ldots \ldots (2.28)
\]

and for the kinematic quantities in Eq. 2.25,

\[
\ddot{x}(t) = \tilde{x}(\omega) e^{i\omega t}
\]

\[
\dot{x}(t) = -i \frac{\tilde{x}(\omega)}{\omega} e^{i\omega t} \quad \ldots \ldots \ldots (2.29)
\]

\[
x(t) = \frac{-i}{\omega^2} \tilde{x}(\omega) e^{i\omega t}
\]
where all are expressed in terms of the complex acceleration response function.

It is now convenient to describe the dynamic pressure on the surface of the cylinder in terms of total structure acceleration, assuming that linear superposition applies, and ground acceleration is
\[ \ddot{u}_g = e^{i\omega t} . \]

\[ P_x(z,\phi,t) = \left[ \overline{P}_{x0}(z,\phi,\omega) + \overline{P}_{x1}(z,\phi,\omega) \bar{X}(\omega) \right] e^{i\omega t} . \]

Replacing the time dependent pressure in Eq. 2.27 with Eq. 2.30, we have:
\[
P^*(t) = \int_0^H \int_0^{2\pi} \overline{P}_{x0}(z,\phi,\omega) R \cos \phi d\phi dz \\
+ \int_0^H \int_0^{2\pi} \overline{P}_{x1}(z,\phi,\omega) R \cos \phi d\phi dz \] e^{i\omega t} . \]

We can now see that the first term on the right of Eq. 2.31 represents the complex hydrodynamic "mass" associated with the rigid motion of the structure on its foundation and the second term represents "mass" associated with the relative motion between the structure and the foundation.

It is more physically relevant to describe these complex "mass" terms as a real virtual mass and a real dampening which is associated with the wave making component of pressure acting on the structure.

The terms in Eq. 2.31 become,
\[
H \int_0^{2\pi} \int_0^{r_{x0}} \overline{P}_{x0}(z,\phi,\omega) R \cos \phi d\phi dz = M^*_x(\omega) - \frac{iC^*_x(\omega)}{X_0(\omega)} . \]

\[
H \int_0^{2\pi} \int_0^{r_{x1}} \overline{P}_{x1}(z,\phi,\omega) R \cos \phi d\phi dz = M^*_x(\omega) - \frac{iC^*_x(\omega)}{X_1(\omega)} . \]
We note that in the case of horizontal motion the terms for rigid and relative motion are the same for a rigid structure on an elastic foundation, however, in general they are not.

Returning to the notation used in Eq. 2.22, we can now describe the hydrodynamic forces of Eq. 2.27 as

\[
F_x(t) = \left( [M_x^* (\omega) + \frac{i \omega C_x^* (\omega)}{\omega} \bar{X}(\omega) - \frac{i}{\omega} C_x^*(\omega) \bar{X}(\omega)] \right) e^{i \omega t} = F_x^* (\omega) e^{i \omega t}
\]

We emphasize that in the general case, the coefficients may be frequency dependent.

\[\text{2.3.3 The Acceleration Response Function}\]

Substituting Eq. 2.33 and Eq. 2.29 into Eq. 2.25, we have

\[
\left[ M_x \ddot{X}(\omega) - \frac{i C_x \bar{X}(\omega)}{\omega} - \frac{K \bar{X}(\omega)}{\omega^2} \right] e^{i \omega t} = \left[ -M_x - \frac{M_x^* (\omega)}{\omega^2} - \frac{M_x^* (\omega)}{\omega} \bar{X}(\omega) + \frac{i C_x^*(\omega)}{\omega} + \frac{i C_x^*(\omega) \bar{X}(\omega)}{\omega} \right] e^{i \omega t} \ldots \ldots \ldots \ (2.34)
\]

This expression can be solved for \(\bar{X}(\omega)\), which represents the acceleration amplification factor for horizontal motion. The solution yields:

\[
\bar{X}(\omega) = \frac{\{M_x (-\omega^2 \bar{X} + K) - C_x \bar{C}\} + i \{\frac{C_x}{\omega} (-\omega^2 \bar{X} + K) - \omega \bar{C} \}}{\omega^2 \bar{X}^2 - 2 K \bar{X} + \frac{K_x^2}{\omega^2} + \bar{C}^2} \ldots \ldots \ldots \ (2.35)
\]

where directional and frequency dependent notation have been dropped in the coefficients and where
\[ N = M_x + M_{xx} \]

and,

\[ C = C_x + C_{xx} \]

2.3.4 The Hydrodynamic Pressure Response Function

We can now describe the hydrodynamic pressure in terms of the acceleration response function as

\[ \tilde{P}^*(\omega) = \{ M^* + M^* \bar{X}_R(\omega) + \frac{C^*}{\omega} X_I(\omega) \} + i\{ M^* \bar{X}_I(\omega) - \frac{C^*}{\omega} \bar{X}_R(\omega) \} \]

\[ = P_R^*(\omega) + i P_I^*(\omega) \]

where \( \bar{X}_R \) and \( \bar{X}_I \) are the magnitudes of the real and imaginary parts of Eq. 2.35, respectively.

We can now describe a convenient expression for the steady-state harmonic pressure force function of Eq. 2.31 in terms of structure system characteristics and hydrodynamic coefficients. Recalling that the response to the real part of the excitation \( e^{i\omega t} \) is the real part of \( \tilde{P}^*(\omega) e^{i\omega t} \) (Eq. 2.33), we can state the harmonic pressure response as

\[ P^*(t) = P_R^*(\omega) \cos(\omega t) - P_I^*(\omega) \sin(\omega t) \]

This expression represents hydrodynamic pressure force per unit of ground acceleration. For the cases where dampening due to wave making can be ignored, this expression in its entirety becomes:

\[ P^*(t) = \left\{ M^* + M^* \frac{-\omega^2 N^2 + \bar{M} \bar{C}}{\omega^2 N^2 - 2\bar{M} \bar{K} + \frac{\bar{K}^2}{\omega^2} + C^2} \right\} \cos(\omega t) \]

\[ + \left\{ \frac{-\omega M^* \bar{N} \bar{C}}{\omega^2 N^2 - 2\bar{M} \bar{K} + \frac{\bar{K}^2}{\omega^2} + C^2} \right\} \sin(\omega t) \]
where,

\[ \bar{M} = \text{total mass as in Eq. 2.35}, \]

\[ M^* = \text{hydrodynamic virtual mass, the real part of Eq. 2.32}, \]

\[ C = \text{foundation damping only}, \]

\[ = 2 \xi_n \bar{\omega}_n M_n \text{ where } \xi_n \text{ is percent of critical damping, } \bar{\omega}_n \text{ is the} \]

natural frequency of the system in that mode, and \( M_n \) is the

dry mass of the structure,

\[ K = \text{foundation stiffness}, \]

\[ \omega = \text{radial frequency of the excitation}. \]

The magnitude of the hydrodynamic pressure force is

\[ |\bar{F}^*(\omega)| = (\bar{P}^*_R + \bar{P}^*_I)^{\frac{1}{2}} \]

(2.40)

and the phase angle relative to ground acceleration is

\[ \phi = \text{ARCTAN} \left( \frac{P^*_I}{P^*_R} \right) \]

(2.41)

It should be noted that an expression for vertical pressure

force can be developed in a similar manner. The differences occur

because the virtual mass and damper terms as described in Eq. 2.32a

and 2.32b are not equal for the vertical case. These differences must

be considered in the pressure response function, Eq. 2.37, and when

developing the acceleration response function, Eq. 2.35. The nature

of the vertical virtual mass terms will be discussed further in Chapter

4.
3. THE SUBMERGED TANK EXPERIMENT

Before proceeding into the details of the submerged tank model and experiment, we shall take a closer look at the reduced equations of motion as stated in Eqs. 2.22 - 2.24. We have previously stated that the coupled terms can be dropped from Eq. 2.22. With this simplification we can rewrite all of the equations for solution:

\[ M^{*} \dddot{x} = -K^{*} \dddot{x} - C^{*} \dddot{x} \]  \hspace{1cm} (3.1)
\[ M^{*} \dddot{z} = -K^{*} \dddot{z} - C^{*} \dddot{z} \]  \hspace{1cm} (3.2)
\[ M^{*} \dddot{\theta} + M^{*} \dddot{x} = -K^{*} \dddot{\theta} - C^{*} \dddot{\theta} \]  \hspace{1cm} (3.3)

3.1 Model Design

The general intent of the model design for the experiment was, simply stated, to be able to measure or at least accurately estimate all of the coefficients and kinematic quantities in Eq. 3.1 - 3.3 which are required for solution for the virtual mass terms in \([M^*]\), while allowing the possibility of varying the foundation stiffness in a controlled manner.

The design was simplified by the fact that the prototype was axisymmetric, therefore requiring only three degrees of freedom in a two-dimensional plane (see Fig. 1.1).

3.1.1 The Elastic Foundation

The single most challenging feature in the model design was to simulate an elastic foundation such that an appropriate relationship could be maintained between horizontal and rotational stiffness and
therefore provide a more realistic framework in which to access the
importance of hydrodynamic coupling and the possible influence of non-
linearities. An elastic half-space formulation for foundation im-
pedances was chosen from which to derive the stiffness coefficients.\textsuperscript{13}
An appropriate mean stiffness value for the frequency range tested was
chosen in each case. APPENDIX A contains the details of the foundation
impedances considerations and the approximations which were necessary
to select appropriate stiffness coefficients. The results of this analy-
sis are shown in Figures 3.1 - 3.3. The approximate equivalent proto-
type stiffnesses for the three foundation conditions examined are
indicated on these figures. It was unnecessary to maintain a consistent
relationship between the vertical stiffness and stiffness in the other
two modes, since no coupling was anticipated with the vertical mode.
The vertical stiffnesses used were somewhat greater than required by
the elastic half-space model for the respective horizontal and rotational
stiffness. This resulted from construction considerations. Elastic
coupling in the foundation was also eliminated so that analysis of hydro-
dynamic coupling would be simpler (see APPENDIX B).

The early stages of foundation design included plans for adding
viscous damping to the foundation in approximate agreement with that
called for by the elastic half-space model. Careful consideration of
the dynamics of the system (see Eq. 2.35) revealed, however, that added
foundation dampening would only mask the effect of hydrodynamic dampen-
ing, if any existed. Therefore, this feature was excluded from the
final model design. The completed model was found to have approximately
0.8% of critical dampening in all three modes in the dry condition, as
will be discussed in Chapter 4.

Control of the foundation stiffness was accomplished by the
design of compound cantilever springs, an example of which is shown in Fig. 3.4. The details of the stiffness analysis of these load cells and their operation in the model are contained in APPENDIX B. As is demonstrated, these cells could be assembled to give an appropriate stiffness in the axial direction and in shear, and produced no rotational coupling when assembled in the model with the model center of gravity adjusted vertically to the midpoint of the four load cells. The load cells were instrumented with full bridge strain gauge rosettes for force measurements in both the axial and shear directions, as shown in Fig. 3.5. Each load cell was calibrated on a dynamometer for its exact stiffness in the two directions and for force-strain relationships. Examples of these calibrations for the three conditions are contained in APPENDIX B. Table 3.1 contains a summary of the mean stiffness values of the three conditions and their approximate equivalent secant stiffness modulus (G/Su) for the prototype system.

Fig. 1.1 shows the model idealization of the prototype system. Fig. 3.6 shows the general arrangement of the load cells on the model base, with typical dimensions indicated.

3.1.2 Model Shell and Instrumentation

The shell of the model was designed to be simple and rigid. The cylindrical portion was rolled from a 25 mm thick aluminum plate. The top and bottom plates were of 19 mm thick aluminum and were fitted with neoprene gaskets to form watertight seals to the cylinder. The bottom gasket extended out from the model and attached to the foundation plate on which the model was mounted. This gasket completed the watertight seal and was flexible enough to allow free movement of the model. The details of this arrangement are shown in Fig. 3.7.
The model shell was suspended on the foundation load cells from the top of the bottom plate, Fig. 3.8, with the upper part of each load cell being attached to an aluminum center post which was in turn attached to the shaking table. All of the model structural parts were reinforced to the fullest extent to hold deformations to a minimum.

The final weight balance in the model was accomplished by attaching lead weights at appropriate locations such that a mass distribution which was considered reasonable for a structure of this nature could be achieved. Table 3.2 shows the final dimensions and weight characteristics of the model and an equivalent prototype on a scale of 1:100.

Four different types of instrumentation were installed in the model:

a. accelerometers for recording total accelerations in the three degrees of freedom of the model and horizontally and vertically on the foundation. Angular acceleration was recorded on the shaking table itself.

b. displacement transducers for relative displacements of the model from the foundation.

c. full bridge strain gauge rosettes on each load cell, one each for horizontal and vertical directions, calibrated to read foundation spring forces directly.

d. pressure sensors in one quadrant, arranged at Gaussian quadrature points for integration of forces.

The general arrangement of the instrumentation is shown in Figs. 3.7 and 3.8. Table 3.3 lists the details and specification for the actual instruments used. The actual model with cover and foundation
support removed is shown in Figs. 3.9 and 3.10.

The model interior was pressurized to an equivalent hydraulic head exceeding the actual water depth by approximately 30 cm. during all tests to protect the instrumentation.

3.2 Experimental Setup and Test Procedures

Testing of the submerged tank model was conducted in two test series on the earthquake simulator, located at the Earthquake Engineering Research Center, Richmond Field Station, University of California, Berkeley. The facility contains a 6-by-6-meter, shaking table with a load carrying capacity of approximately 60 metric tons. The table can be excited harmonically either horizontally, vertically, or both simultaneously at frequencies up to approximately 30 Hz. It has a maximum horizontal stroke amplitude of approximately 15 cm and can achieve accelerations of approximately 1 g (g = 980.7 cm/sec^2) within the stroke limitation. Random excitations can be induced in the table from magnetic tape input. The facility has direct recording capabilities for 128 digitized data channels to computer compatible magnetic tape.

A rigid bulkhead was constructed to surround the shaking table, and a flexible membrane was placed over the table and bulkhead to form the test basin. Figure 3.11 shows the experiment arrangement and Fig. 3.12 shows the basin and model as filling begins.

The test procedure for each foundation condition and water depth was approximately as follows:

a. shock tests for natural frequencies,

b. vertical and horizontal harmonic tests over the range of frequencies from 3 to 19 Hz, using at least two different values of
acceleration in most cases, up to a maximum of about 0.5 g when conditions allowed,

c. vertical and horizontal random excitation with maximum acceleration of approximately 0.3 g.

This procedure was also carried out for the dry condition for calibration purposes.

Four water depths were tested in this study, ranging from level with the model top to a maximum of 2.5 times the height of the model, a depth of 85 centimeters.

3.3 Data Analysis

Referring to Eq. 3.1 - 3.3, the tests yielded the following information in our effort to solve for the virtual mass terms:

a. direct measurement of the total structure acceleration, \([\ddot{r}_t]\),

b. direct measurement of foundation acceleration, \([\ddot{u}_g]\),

c. direct measurement of structure relative displacement, \([r]\),

d. direct measurement of structure mass, \([M]\),

e. determination of material damping in each mode from the dry shock tests, \([C]\),

f. direct measurement of foundation stiffness, \([K]\),

g. determination of damping due to hydrodynamic effects from the submerged shock tests, \([C^*]\).

The only remaining quantities needed in order to proceed with the solution were the velocities, and these were calculated from the acceleration and displacement time series using numerical integration and differentiation schemes.
In theory Eqs. 3.1 and 3.2 could be solved at any point in the response time series for $M_{xx}$ and $M_{zz}$, since they are satisfied for all time and regardless of the nature of the motion. In fact, linear regression techniques must be applied using a large number of data points before reliable results can be achieved. Multiple regression was used in solving Eq. 3.3 for $M_{yy}$ and $M_{yx}$.

The details of the time series analysis and virtual mass calculations are contained in APPENDIX C. The computer program used to convert the raw experimental data to model response is listed in APPENDIX D. The program used to analyze the model response and calculate the virtual mass is listed in APPENDIX E.
### TABLE 3.1: MODEL FOUNDATION CONDITION SUMMARY (MODEL UNITS)

<table>
<thead>
<tr>
<th>COND. NO.</th>
<th>$K_x$ (N/m)</th>
<th>$K_z$ (N/m)</th>
<th>$K_θ$ (N-m/RAD)</th>
<th>APPROX. PROTOTYPE SECANT MODULUS (G/Su)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>HORIZ. &amp; ROT</td>
</tr>
<tr>
<td>1</td>
<td>$2.9 \times 10^6$</td>
<td>$5.2 \times 10^6$</td>
<td>$3.2 \times 10^5$</td>
<td>2500</td>
</tr>
<tr>
<td>2</td>
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<td>$4.8 \times 10^6$</td>
<td>$2.1 \times 10^5$</td>
<td>1600</td>
</tr>
<tr>
<td>3</td>
<td>$5.3 \times 10^5$</td>
<td>$1.5 \times 10^6$</td>
<td>$5.2 \times 10^4$</td>
<td>500</td>
</tr>
</tbody>
</table>

### TABLE 3.2: SUBMERGED TANK MODEL DIMENSIONS AND CHARACTERISTICS

<table>
<thead>
<tr>
<th></th>
<th>MODEL</th>
<th>PROTOTYPE (SCALE 1:100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEIGHT ($H$)</td>
<td>34.3 cm</td>
<td>34.3 m</td>
</tr>
<tr>
<td>DIAMETER ($D$)</td>
<td>80.3 cm</td>
<td>80.3 m</td>
</tr>
<tr>
<td>C.G. HEIGHT</td>
<td>13.4 cm</td>
<td>13.4 m</td>
</tr>
<tr>
<td>MASS</td>
<td>249.8 kg</td>
<td>249,800 TONS</td>
</tr>
<tr>
<td>RADIUS OF GYR.</td>
<td>26.2 cm</td>
<td>26.2 m</td>
</tr>
<tr>
<td>CONSTRUCTION</td>
<td>MACHINED ALUMINUM</td>
<td>RIGID</td>
</tr>
</tbody>
</table>
TABLE 3.3: INSTRUMENTATION SPECIFICATIONS

<table>
<thead>
<tr>
<th>TYPE</th>
<th>MANUFACTURE</th>
<th>MODEL</th>
<th>RANGE AS USED</th>
<th>APPROX. ACCURACY</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACCELEROMETERS</td>
<td>STATHAM</td>
<td>A39TC-5-350</td>
<td>± 2.5g</td>
<td>1.0%</td>
</tr>
<tr>
<td>DISPLACEMENT TRANSDUCERS</td>
<td>HEWLETT-PACKARD</td>
<td>a) 7DCDT-500</td>
<td>± 1.270 cm.</td>
<td>0.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b) 7DCDT-100</td>
<td>± 0.254 cm.</td>
<td>0.5%</td>
</tr>
<tr>
<td>FORCE TRANSDUCERS (HORIZ. AND VERT.)</td>
<td></td>
<td>*</td>
<td>± 1000 N</td>
<td>± 2% (STATIC)</td>
</tr>
<tr>
<td>HYDRODYNAMIC PRESSURE TRANSDUCERS</td>
<td>SUNDSTRAND DATA CONTROL, INC.</td>
<td>206</td>
<td>± 2 PSI</td>
<td>0.8%</td>
</tr>
<tr>
<td>ANALOG TO DIGITAL CONVERTER</td>
<td>NEFF</td>
<td>SYSTEM-620</td>
<td>128 CHANNEL</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

* MANUFACTURED AT THE UNIVERSITY OF CALIFORNIA, BERKELEY, AS PER CHAPTER 3 AND APPENDIX B.
FIGURE 3.1: PROTOTYPE HORIZONTAL FOUNDATION STIFFNESS ($k_{xx}^b$) VERSUS SOIL SHEAR MODULUS ($G$)
Figure 3.2: Prototype Foundation Stiffness $(K^b_{zz})$ versus Soil Shear Modulus $(G)$. The graph shows data for three conditions: Very Stiff (COND. 1), Soft (COND. 2), and Soft (COND. 3). The x-axis represents the prototype foundation vertical stiffness $(K^b_{zz})$ in kN/m, and the y-axis represents the soil shear modulus $(G)$ in kN/m$^2$. The data points are marked with symbols: * for Very Stiff, △ for Soft, and □ for Soft.
FIGURE 3.3: PROTOTYPE ROTATIONAL FOUNDATION STIFFNESS ($K_{bb}$) VERSUS SOIL SHEAR MODULUS
FIGURE 3.4: A TYPICAL FOUNDATION SPRING
FIGURE 3.5: A FOUNDATION SPRING INSTRUMENTED AS A LOAD CELL
FIGURE 3.7: INSTRUMENTATION ARRANGEMENT
FIGURE 3.8: MODEL INTERNAL ARRANGEMENT
FIGURE 3.10: THE SUBMERGED TANK MODEL, TOP VIEW
4. THE EXPERIMENTAL RESULTS

4.1 General Model Response

The resonant response for the model system in the dry state and at each of the four water depths tested is shown in Table 4.1. The record of resonant response for foundation condition No. 3 and relative water depth \( h/H \) equal 2.5 is shown in Fig. 4.1. It can be seen here that there is no detectable interference between the horizontal and rotational modes of oscillation when they are excited simultaneously. This figure is typical of all of the resonant data recorded. The conclusion to be drawn from this result is that hydrodynamic coupling is not strong, if it exists, and that we were justified in dropping the coupled term from the analysis of horizontal virtual mass in Eq. 3.1. Table 4.2 contains a summary of the dampening, expressed as a percent of critical dampening, derived from the model resonant response data. These values were found using the following free-vibration decay relationship:

\[
\xi = \frac{\delta}{2\pi m} \quad \ldots \ldots \ldots \quad (4.1)
\]

where, 
\( m \) = total number of cycles
\( \delta = \ln \left( \frac{a_n}{a_{n+m}} \right) \), when \( a_n \) is the amplitude at time \( n \) and \( a_{n+m} \) the amplitude \( m \) cycles later.

This expression is exact to within the accuracy of the recorded data (approximately \( \pm 2\% \)).

We use the notation \( \xi^* \) when referring to hydrodynamic dampening in calculations. However, in most cases this is taken to be zero.