4.2 Virtual Mass

The results of the analysis for virtual mass in the horizontal mode are shown in Figs. 4.2 - 4.5, vertical mode in Figs. 4.6 - 4.8, rotational mode in Figs. 4.9 - 4.12, and the horizontal-rotational coupled mass in Figs. 4.13 - 4.16.

The virtual mass in these figures has been plotted versus the dimensionless frequency parameter \( \omega \sqrt{\Omega / \omega} \) which represents the ratio of inertia force to wave generation force in the horizontal mode, as discussed in Chapter 2.

The mass values have been normalized in the following manner:

\[
\begin{align*}
C_{XX}^m &= M_* / (\rho V) \quad \ldots \ldots \quad (4.2a) \\
C_{ZZ}^m &= M_* / (\rho V) \quad \ldots \ldots \quad (4.2b) \\
C_{\theta \theta}^m &= M_* / (\rho V R_g^2) \quad \ldots \ldots \quad (4.2c) \\
C_{\phi X}^m &= M_* / (\rho V R_g) \quad \ldots \ldots \quad (4.2d)
\end{align*}
\]

where,

\( V \) = displaced volume of the structure
\( = \pi R_g^2 H \)

\( R_g \) = radius of gyration of the displaced volume of water about its center of gravity

\( \rho \) = mass density of fresh water

The results from the diffraction theory calculations by Garrison for the relative water depth \( h/H = 2.5 \) and from the closed-form solution after Liaw for \( h/H = 1.0 \) have been included on the appropriate figures.

Our first calculation of the vertical coefficient yielded a value of approximately 1.0 at low frequencies and converged to the
theory at the natural frequencies of the various foundation conditions. This implied that there was a basic error in our formulation, since one would not expect to see frequency dependence in the coefficients which was related so closely to structure response.

This result has caused us to take a closer look at the physical system we are modeling with Eq. 2.1. We are implying by our use of a single inertia coefficient with the sum of the relative acceleration and the foundation acceleration that each of these kinematic conditions excites the same flow regime above the structure. To put it somewhat differently, we are saying that relative motion without foundation motion and foundation-structure motion without relative motion excite the same flow conditions. If one thinks in terms of an infinite rigid foundation and an incompressible fluid it is apparent that this is not true. In the latter case the structure would feel the effect of the entire mass of the water column above it during foundation vertical accelerations. This situation is shown graphically in Fig. 4.17.

We can describe the forces due to vertical ground acceleration as the summation of the force due to rigid body motion of the structure-foundation system (no relative motion)

\[
F_z' = (M_z + \rho \Delta H \frac{\pi D^2}{4}) \dddot{u}_g
\]

and the force due to the relative motion of the structure above

\[
F_z'' = (M_z + \rho C_{zz}^m H \frac{\pi D^2}{4}) \dddot{z}
\]

with the total force being

\[
F_z = F_z' + F_z''
\]
We must now rewrite Eq. 3.2 to solve for the vertical virtual mass as follows:

\[ M_v \ddot{z} = -k_v \ddot{z} - \rho g H^2 R \ddot{y} - C_v \dot{z} - k_v z - C_v^{\ddot{z}} \ldots \ldots (4.3) \]

The vertical mass coefficients shown have resulted from use of this formulation.

4.3 Hydrodynamic Pressure Force

Hydrodynamic pressure forces were recorded in some of the harmonic tests for comparison with the pressure force which would be predicted using the calculated virtual masses and Eq. 2.40, as discussed in Section 2.3.4.

For comparison purposes we have calculated the magnitude of the hydrodynamic force using Eq. 2.39 and the mean virtual mass values for horizontal and vertical modes. These results for each depth are shown in Figs. 4.18 - 4.24 plotted as a function of the dimensionless frequency parameter. All values shown are normalized to the \( 1g \) (980.7 cm/sec²) foundation acceleration level.

Also shown on these figures are the actual magnitude of the pressure force amplitudes recorded in the testing. These forces were derived from forces measured on one quarter of the structure, as shown in APPENDIX C. The plotted values represent the average force amplitudes observed, generally taken for a minimum of 30 cycles.

Hydrodynamic and structural dampening were both very small in the model system, ranging between approximately 0.8 and 3.0 percent of critical depending on the condition. This results in the pressure force being nearly in phase with foundation acceleration and essentially real valued, except near resonance. We have, therefore, not
presented data on the pressure force phase relationship observed in the testing. We will discuss the effects of foundation dampening on the phase angle in Chapter 5 using calculated values based on Eq. 2.39.
### TABLE 4.1: MODEL RESONANT RESPONSE FREQUENCIES (Hz)

<table>
<thead>
<tr>
<th>FOUNDATION CONDITION</th>
<th>RELATIVE WATER DEPTH (h/H)</th>
<th>0 (DRY)</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
<td>17.1</td>
<td>15.4</td>
<td>14.8</td>
<td>14.6</td>
<td>14.6</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>22.9</td>
<td>22.9</td>
<td>21.0</td>
<td>19.3</td>
<td>18.9</td>
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<tr>
<td></td>
<td>θ</td>
<td>21.7</td>
<td>21.5</td>
<td>20.0</td>
<td>19.9</td>
<td>19.8</td>
</tr>
<tr>
<td>2</td>
<td>X</td>
<td>13.9</td>
<td>12.5</td>
<td>12.1</td>
<td>12.0</td>
<td>11.9</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>22.1</td>
<td>22.1</td>
<td>20.3</td>
<td>18.6</td>
<td>18.2</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td>17.6</td>
<td>17.2</td>
<td>16.4</td>
<td>16.2</td>
<td>16.2</td>
</tr>
<tr>
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<td>X</td>
<td>7.3</td>
<td>6.6</td>
<td>6.5</td>
<td>6.4</td>
<td>6.3</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>12.3</td>
<td>12.3</td>
<td>11.0</td>
<td>10.6</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td>8.8</td>
<td>8.3</td>
<td>8.1</td>
<td>8.0</td>
<td>8.0</td>
</tr>
</tbody>
</table>

### TABLE 4.2: MODEL DAMPING SUMMARY (PERCENT OF CRITICAL)

<table>
<thead>
<tr>
<th>FOUNDATION CONDITION</th>
<th>MODE</th>
<th>RELATIVE WATER DEPTH (h/H)</th>
<th>0 (DRY)</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
<td></td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td></td>
<td>0.8</td>
<td>0.9</td>
<td>3.0</td>
<td>2.4</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td></td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>X</td>
<td></td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td></td>
<td>1.4</td>
<td>1.3</td>
<td>1.6</td>
<td>4.0</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td></td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
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<td></td>
<td>1.9</td>
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<td>1.9</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td></td>
<td>1.2</td>
<td>0.9</td>
<td>0.7</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td></td>
<td>1.6</td>
<td>1.4</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>
FIGURE 4.1: UNCOUPLED RESONANT RESPONSE OF THE MODEL, $h/H = 2.5$
Figure 4.2: Horizontal inertia coefficient, $h/H = 2.5$

- Found conditions:
  - Very stiff
  - Stiff
  - Soft

Data mean: 0.517
Std. dev.: 0.047
Nr. pts.: 74
FIGURE 4.3: HORIZONTAL INERTIA COEFFICIENT, h/H = 2.0
FIGURE 4.4: HORIZONTAL INERTIA COEFFICIENT, $h/H = 1.5$

- DATA MEAN: 0.465
- STD. DEV.: 0.043
- NR. PTS.: 42
FIGURE 4.5: HORIZONTAL INERTIA COEFFICIENT, $h/H = 1.0$
Figure 4.6: Vertical Inertia Coefficient, h/H = 2.5
Figure 4.7: Vertical Inertia Coefficient, $h/H = 2.0$
Figure 4.8: Vertical Inertia Coefficient, h/H = 1.5
FIGURE 4.9: ROTATIONAL INERTIA COEFFICIENT, h/H = 2.5
FIGURE 4.10: ROTATIONAL INERTIA COEFFICIENT, h/H = 2.0
h/H = 1.5

DATA MEAN: 0.1030
STD. DEV.: 0.501
NR. PTS.: 36

FOUND. COND. △ VERY STIFF
◇ STIFF
◇ SOFT

\[
\frac{\omega^2 \sqrt{DH}}{g}
\]

\(C_m\)
**Figure 4.12: Rotational Inertia Coefficient, \( h/H = 1.0 \)**

- **h/H = 1.0**
- Data Mean: -0.0778
- Std. Dev.: 0.3363
- N.R. Pts.: 28

Found conditions:
- \( \triangle \) Very Stiff
- \( \Diamond \) Stiff
- \( \Diamond \) Soft

Data mean line graph with values of \( \frac{\omega^2 DH}{g} \) on the x-axis ranging from 10 to 5000, and \( C_m \) on the y-axis ranging from -4.0 to 4.0.
FIGURE 4.13: COUPLED INERTIA COEFFICIENT, h/H = 2.5
Figure 4.14: Coupled Inertia Coefficient, \( h/H = 2.0 \)
Figure 4.15: Coupled Inertia Coefficient, h/H = 1.5
FIGURE 4.16: COUPLED INERTIA COEFFICIENT, $h/H = 1.0$
(1) FOUNDATION EXCITATION ONLY

\[ F_w' = \rho \Delta H \pi D^2 \frac{V_g}{4} \]

- INCOMPRESSIBLE FLUID (INFINITE)
- RIGID STRUCTURE
- RIGID SEA BED
- RIGID FOUNDATION

\[ F_s' = M_z \ddot{V}_g \]

FORCE DUE TO RIGID BODY MOTION

(2) RELATIVE MOTION ONLY

\[ F_w'' = \rho C_{zz}^m H \pi D^2 \ddot{Z} \]

- RIGID STRUCTURE
- MOTIONLESS SEA BED
- ELASTIC FOUNDATION

\[ F_s'' = M_z \ddot{Z} \]

FORCE DUE TO RELATIVE MOTION

\[ F_z' = (M_z + \rho \Delta H \pi D^2 \frac{V_g}{4}) \ddot{V}_g = \]

\[ F_z'' = (M_z + \rho C_{zz}^m H \pi D^2) \ddot{Z} = \]

\[ F_z = M_z (\ddot{Z} + \ddot{V}_g) + \rho C_{zz}^m H \pi D^2 \ddot{Z} + \rho \Delta H \pi D^2 \ddot{V}_g \]

\[ F_z = M_z (\ddot{Z} + \ddot{V}_g) + \rho C_{zz}^m H \pi D^2 \ddot{Z} + \rho \Delta H \pi D^2 \ddot{V}_g \]

FIGURE 4.17: VERTICAL INERTIA FORCE REPRESENTATION
FIGURE 4.18: CALCULATED VERSUS MEASURED HORIZONTAL HYDRODYNAMIC FORCE AMPLITUDE, $h/H = 2.5$

\begin{align*}
\frac{p^*_x}{\rho V_g} & \quad \text{(HYDRODYNAMIC FORCE AMPL.)} \\
\frac{\omega^2 \sqrt{D_H}}{g} & \quad \text{(FREQUENCY)}
\end{align*}

- $\Delta$ CALCULATED (EQ. 2.37)
- $C_m^x = 0.520$
- $\xi_x^* = 0$
- $\xi_x = 0.8\%$
- $\times$ - MEASURED

VERY STIFF FOUND.

$\ddot{u}_g = 1g$
\[ \frac{h}{H} = 2.0 \]

VERY STIFF FOUND.

\[ \ddot{u}_g = \dot{u}_g \]

\[ \begin{align*}
\Delta \text{ CALCULATED (EQ. 2.37)} \\
C_{xx}^m &= 0.500 \\
\xi^* &= 0 \\
\xi_x &= 0.8 \% \\
\times &\text{ MEASURED}
\end{align*} \]

**FIGURE 4.19:** CALCULATED VERSUS MEASURED HORIZONTAL HYDRODYNAMIC FORCE AMPLITUDE, \( \frac{h}{H} = 2.0 \)
**Figure 4.20:** Calculated versus measured horizontal hydrodynamic force amplitude, \( h/H = 1.5 \)

- \( h/H = 1.5 \)
- Very stiff found.
- \( \bar{u}_g = 1g \)

**Equation 2.37**

- \( C_{xx}^m = 0.465 \)
- \( \xi_x^* = 0 \)
- \( \xi_x = 0.8\% \)
- * - Measured
$\Delta$ CALCULATED (EQ 2.37)

$C_m^{xx} = 0.340$

$\xi_x^* = 0$

$\xi_x = 0.8\%$

* - MEASURED

FIGURE 4.21: CALCULATED VERSUS MEASURED HORIZONTAL HYDRODYNAMIC FORCE AMPLITUDE, $h/H = 1.0$

$\frac{\omega^2 \sqrt{DH}}{g}$

HYDRODYNAMIC FORCE AMPL. $p_x^*/p_v g$

$h/H = 1.0$

VERY STIFF FOUND.

$\ddot{u}_g = 1g$
**Figure 4.22: Calculated Versus Measured Vertical Hydrodynamic Force Amplitude, h/H = 2.5**

- **h/H = 2.5**
- **Very stiff found.**
- **V̇_g = 1g**
- **C_m = 0.630**
- **ξ_x = 0**
- **ε_x = 0.8%**
- **- Measured**
\( \frac{h}{H} = 2.0 \quad \triangle \text{CALCULATED (EQ 2.37)} \)

VERY STIFF FOUND.

\[ C_{xx}^m = 0.590 \]

\[ \dot{V}_g = 1 \text{g} \]

\[ \xi_x = 0 \]

\[ \xi_x = 0.8\% \]

\[ \star \text{- MEASURED} \]

\[ \frac{\omega^2 \sqrt{D}}{g} \]

\( \frac{P_2}{\rho V_g} \)

**Figure 4.23:** Calculated versus measured vertical hydrodynamic force amplitude, \( h/H = 2.0 \)
Figure 4.24: Calculated versus measured vertical hydrodynamic force amplitude, h/H = 1.5

- \( h/H = 2.5 \)
- \( \Delta \) Calculated (Eq. 2.37)
- Very stiff found.
  - \( C_{zz}^m = 0.400 \)
  - \( \xi_z^* = 0 \)
  - \( \xi_z = 0.8\% \)
  - * - Measured
5. SUMMARY AND DISCUSSION

5.1 Inertia Coefficients

Summaries of the inertia coefficients resulting from these experiments are shown in Figs. 5.1 - 5.4 as a function of relative water depth (h/H). The plotted values are the average of all data taken in each condition, as presented in Chapter 4.

5.1.1 Comparison with Theoretical Values

Table 5.1 shows a comparison of the data average values with those predicted by the two analytical techniques considered in this study. All averages have been taken over the range of the dimensionless frequency parameter (o) from 20 to 1000.

It can be seen that the agreement between the measured and predicted values is quite good. In all cases, the differences are less than nine percent (9%).

These results are somewhat surprising in some cases, e.g., the rotational coefficients of Fig. 4.9, in light of the large standard deviation of data values. The scatter in these data is related to the fact that the rotational acceleration observed in the tests was very small. However, the numbers of samples observed was sufficiently large to give a good approximation of the mean value of quantities which are essentially constant. This is apparently the case with the rotational inertia terms. It is interesting to observe that both the horizontal and vertical inertia terms decrease rapidly for h/H less than about 2.0, and they appear to approach a constant value for relative water depths greater than 2.0.
5.1.2 Frequency Dependence

Examination of the experimental results and theoretical values shows that frequency dependence in the inertia coefficients is generally very small for \( \sigma \) greater than 20.

The only appreciable change with frequency occurs in horizontal motion for the case of the structure piercing the surface \( (h/H = 1.0) \), Fig. 4.5. In this case, the coefficient is essentially constant for values of \( \sigma \) greater than about 50. For smaller values of this parameter, the inertia term decreases rapidly with decreasing frequency. It should be noted that the theoretical solution in this case includes the effect of wave generation by the structure. The force dissipated in wave generation exceeds one percent \((1\%)\) of the inertia force at about \( \sigma \) equal 20 and increases rapidly for lower frequencies.

Ignoring frequency dependence in the surface piercing case would mean that the inertia coefficient would be overestimated by approximately thirteen percent \((13\%)\) at \( \sigma \) equal 20. This error would be expected to decrease rapidly with submergence of the structure.

5.1.3 Horizontal-Rotational Coupling

Coupling between the horizontal and rotational modes of motion in the inertia coefficients is predicted by the theory and can be measured, Fig. 4.13. However, the value of this coupling is very small.

Eigenvalue calculations with the largest value of hydrodynamic coupling observed in these tests show that the effect is seen in the
fifth digit of natural frequency and mode shape.

5.2 Hydrodynamic Pressure Force

Most of the considerations in this report have dealt with defining the hydrodynamic inertia coefficients. We would now like to consider the variation in hydrodynamic pressure force directly before we conclude our study.

5.2.1 Parameter Sensitivity

We have stated previously that the hydrodynamic pressure force can be related directly to structure response (and vice versa) through the inertia terms, e.g., Eqs. 2.37 and 2.39. We would now like to examine which of the system characteristics most affect the pressure force.

Fig. 5.5 shows the effect of a fifteen percent (15%) increase or decrease in the value of the horizontal inertia coefficient. These results show that the pressure force changes approximately in proportion to the change in the inertia coefficient (actually + 17%). In addition, there is the resonant frequency shift that would be expected.

Fig. 5.6 shows the effect of a ten percent (10%) increase or decrease in foundation stiffness. It can be observed that foundation stiffness changes only shift the location of resonance and have no influence on the magnitude of the pressure force.

Fig. 5.7 shows the effect of change in foundation dampening from approximately eight tenths of one percent (0.8%) of critical, the value observed in the model, to fifteen percent (15%). We note that
the pressure force decreases by sixty-five percent (65%) as foundation dampening is increased from five percent (5%) to fifteen percent (15%) of critical.

5.2.2 Phase Angle

Fig. 5.8 shows the phase relationship between hydrodynamic pressure force and foundation acceleration for varying values of foundation dampening. The phase angle is seen to be highly dependent on foundation dampening at frequencies near resonance. It is also interesting to note that while the pressure force and foundation acceleration are in phase at frequencies far below resonance, they are out of phase by a fixed angle dependent on the damping at frequencies far above resonance.

5.3 Foundation forces in Random Excitation

We have examined fluid-structure interaction in harmonic motion in considerable detail and have demonstrated that hydrodynamic inertia forces can be measured or predicted accurately under these conditions.

However, earthquakes occur as random ground accelerations and structures must be designed to withstand this condition.

One common method of determining response to random excitation is step-by-step integration of the equations of motion using a discrete acceleration time series as the forcing function. A variety of methods are available to accomplish this calculation. Most of these methods rely on system coefficients which are constant in frequency and independent of the magnitude of structure response.
The hydrodynamic coefficients measured in these experiments meet the requirements for use in step-by-step integration. Therefore, we can use this technique to examine the differences between forces measured during actual random excitations and those that can be calculated using a digitized record of the same ground acceleration.

The above comparison has been performed using the program SUBTANK which is listed in APPENDIX F. This program is based on integration methods and subroutines developed by Professor E. L. Wilson of the Civil Engineering Department at the University of California, Berkeley. The ground acceleration used in testing was a reproduction of the N-S component of the 1940 El Centro earthquake, scaled to a maximum acceleration value of approximately 0.31 g (304 cm/sec$^2$). A version of this record was used as a control signal for the earthquake simulator table, and the resulting table and structure responses were recorded. A digitized record of the actual table acceleration was then used with the horizontal inertia coefficient determined by Garrison, as previously discussed, to calculate a foundation shear force time history.

Fig. 5.9(a) shows a plot of the horizontal foundation acceleration recorded (and used in the step-by-step integration) and Fig. 5.9(b) shows the measured horizontal shear force between the structure and foundation. Fig. 5.9(c) shows the calculated shear force time history assuming hydrodynamic damping equals two tenths of one percent (0.2%) of critical and foundation damping equals eight tenths of one percent (0.8%), the values determined from the resonant decay tests. Coupling between horizontal and rotational modes has been neglected in this calculation. The case considered is for horizontal ground acceleration only.
The maximum shear force measured in this test run was 3745 Newtons compared to a calculated force of 3740 Newtons, for a difference of approximately one tenth of one percent (-0.1%). The conclusion to be drawn is that the hydrodynamic effects are very linear and can be properly considered by use of constant coefficients.

Fig. 5.10 shows three additional calculated horizontal shear force time histories using the ground acceleration record of Fig. 5.9(a).

Fig. 5.10(a) shows the force record achieved when hydrodynamic dampening is ignored. The maximum shear force calculated was 3834 Newtons, an increase of approximately two and four tenths percent (2.4%) over the measured value. This would not be an important increase in most applications. However, one can see from the effect of this small amount of dampening that it would only need to be a little greater before the resulting force reduction would begin to be significant. This would occur as the depth of submergence was decreased.

Figs. 5.10(b) and 5.10(c) show horizontal shear force time histories calculated for foundation dampening of five percent (5%) and fifteen percent (15%) of critical, respectively. The maximum shear force is seen to increase by approximately fifty percent (≈50%) for a decrease in foundation dampening over this range. It is apparent that foundation dampening will be a major consideration in properly predicting foundation forces induced by earthquakes.
<table>
<thead>
<tr>
<th>COEFFICIENT</th>
<th>h/H</th>
<th>PREDICTED</th>
<th>MEASURED</th>
<th>DIFFERENCE</th>
</tr>
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<td>0.52</td>
<td>4 %</td>
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<tr>
<td>HORIZONTAL</td>
<td>1.0</td>
<td>0.34</td>
<td>0.34</td>
<td>~1 %</td>
</tr>
<tr>
<td>VERTICAL</td>
<td>2.5</td>
<td>0.64</td>
<td>0.62</td>
<td>3 %</td>
</tr>
<tr>
<td>ROTATIONAL</td>
<td>2.5</td>
<td>0.26</td>
<td>0.24</td>
<td>8 %</td>
</tr>
<tr>
<td>COUPLED</td>
<td>2.5</td>
<td>0.037</td>
<td>0.036</td>
<td>3 %</td>
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FIGURE 5.1: HORIZONTAL INERTIA COEFFICIENT ($c_{xx}^m$) MEAN VALUES VERSUS RELATIVE DEPTH ($h/H$)
FIGURE 5.2: VERTICAL INERTIA COEFFICIENT \( c_{zz}^m \) MEAN VALUES VERSUS RELATIVE DEPTH \((h/H)\)
Figure 5.3: Rotational Inertia Coefficient ($c_{\theta \theta}^{m}$) Mean Values Versus Relative Depth ($h/H$)
FIGURE 5.4: COUPLED INERTIA COEFFICIENT ($c^n_{6x}$) MEAN VALUES VERSUS RELATIVE DEPTH ($h/H$)
$h/H = 2.5$

VERY STIFF FOUND.

$\xi_x^* = 0.2\%$

$\xi_x = 0.8\%$

$\ddot{u}_g = 1g$

CALCULATED (EQ. 2.37)

- $C_{xx}^m = 0.60$
- $C_{xx}^m = 0.52$
- $C_{xx}^m = 0.44$

FIGURE 5.5: VARIATION IN HYDRODYNAMIC FORCE WITH CHANGES IN VIRTUAL MASS
Figure 5.6: Variation in hydrodynamic force with changes in foundation stiffness.

Varying stiffness found:
- $h/H = 2.5$
- $C_x = 0.520$
- $\xi_x = 0.8\%$
- $\beta = 1g$

Hydrodynamic force amplification $P_0^x g$
CALCULATED (EQ. 2.37)

\* $\xi_x = 0.8\%$
\△ $\xi_x = 5\%$
◇ $\xi_x = 10\%$
◇ $\xi_x = 15\%$

$\frac{h/H = 2.5}{\text{VERY STIFF FOUND.}}$

$C_{xx}^m = 0.520$

$\xi_x^* = 0.2\%$

$\bar{u}_g = 1g$

FIGURE 5.7: VARIATIONS IN HYDRODYNAMIC FORCE WITH CHANGES IN FOUNDATION DAMPING
FIGURE 5.8: HYDRODYNAMIC FORCE PHASE ANGLE RELATIVE TO FOUNDATION ACCELERATION FOR VARIOUS VALUES OF FOUNDATION DAMPENING
Figure 5.9: Comparison of measured and calculated horizontal shear force in the model for the El Centro (1940) earthquake, \( h/H = 2.5 \).
Figure 5.10: Calculated horizontal shear force in the model for the El Centro (1940) earthquake, (a) neglecting hydrodynamic dampening, (b) with foundation dampening equal to 5% of critical, (c) with foundation dampening equal to 15% of critical.
6. CONCLUSIONS

The findings of this study concerning the earthquake response of large gravity-type offshore structures are summarized as follows:

(a) Available analytical techniques provide good estimates of hydrodynamic inertia force coefficients in the range of frequencies of interest for the simple structure configuration considered.

(b) Foundation dampening is a major consideration in determining the magnitude of the hydrodynamic pressure force and the resulting foundation force. The sensitivity of foundation force to foundation dampening indicates that this site characteristic might dominate the design and placement of large offshore structures.

(c) Foundation stiffness only influences the hydrodynamic force by changing the resonant frequency. This characteristic does not influence the magnitude of this force directly.

(d) Frequency dependence in the inertia coefficients is not likely to be an important consideration.

(e) Coupling in the hydrodynamic inertia forces between the horizontal and rotational modes is not likely to be important at earthquake frequencies.

(f) Hydrodynamic dampening will not be an important factor in the earthquake response of deeply submerged structures, but may be significant in near surface and surface-piercing structures.
REFERENCES


APPENDIX A
RESPONSE OF ELASTIC FOUNDATIONS

A1. Complex Foundation Impedance Representation

This study has concerned itself with the rigid body response of large gravity-type structures on elastic foundations. Figure A1 shows such a structure in an exaggerated displaced configuration. The foundation-structure interaction of such a system has been described by a number of researchers. In the case of structures which essentially sit on the bottom, the force-displacement relations of the system can be equated to those of a rigid massless disk resting on a homogeneous foundation. These relationships are expressed in the form of complex frequency dependent functions, the real part of which represents foundation stiffness, and the imaginary part damping. These functions relate a set of harmonic forces, see Fig. A3, applied to the rigid disk at frequency \( \omega \) to the resulting displacements.

The three degree of freedom system subjected to the harmonic forces

\[
\begin{bmatrix}
F_x^b(t) \\
F_z^b(t) \\
F_\theta^b(t)
\end{bmatrix} =
\begin{bmatrix}
F_x^b(\omega) \\
F_z^b(\omega) \\
F_\theta^b(\omega)
\end{bmatrix} e^{i\omega t} \quad \text{A1.1}
\]

has the following force-displacement relations:

\[
\begin{bmatrix}
\ddot z_x^b(\omega) \\
\ddot z_z^b(\omega) \\
\ddot z_\theta^b(\omega)
\end{bmatrix} e^{i\omega t} =
\begin{bmatrix}
-k_{xx}^b & 0 & -k_{x\theta}^b \\
0 & -k_{zz}^b & 0 \\
-k_{\theta x}^b & 0 & -k_{\theta \theta}^b
\end{bmatrix}
\begin{bmatrix}
\ddot z_x^b(\omega) \\
\ddot z_z^b(\omega) \\
\ddot z_\theta^b(\omega)
\end{bmatrix} e^{i\omega t} \quad \text{A1.2}
\]
where \( F^b_x(\omega), F^b_z(\omega), \) and \( F^b_0(\omega) \) are, respectively the harmonic exciting forces and moment at frequency \( \omega \) acting on the rigid massless disk; \( \tilde{x}^b(\omega), \tilde{z}^b(\omega), \) and \( \tilde{\theta}^b(\omega) \) are, respectively, the corresponding harmonic horizontal, vertical, and angular displacements of the base. It should be noted that for a linear system response to a real excitation will also be real valued (see Section 2.3.2). Bars on the above quantities indicate complex values for the general case.

The foundation impedances may be written in the form

\[
\begin{bmatrix}
    -K^b_{xx} & 0 & \tilde{K}^b_{x0} \\
    0 & -K^b_{zz} & 0 \\
    -K^b_{0x} & 0 & -K^b_{00}
\end{bmatrix}
= \begin{bmatrix}
    K^b_{xx} & 0 & K^b_{x0} \\
    0 & K^b_{zz} & 0 \\
    K^b_{0x} & 0 & K^b_{00}
\end{bmatrix}
+ i\omega
\begin{bmatrix}
    -C^b_{xx} & 0 & \tilde{C}^b_{x0} \\
    0 & -C^b_{zz} & 0 \\
    -C^b_{0x} & 0 & -C^b_{00}
\end{bmatrix}
\]

where \( K^b_{ij} \) and \( C^b_{ij} \) terms represent the magnitude of stiffness and dampening, respectively, in the various modes.

It has been noted that the coupling between horizontal and rotational motion of the rigid disk is negligible\(^{13,19}\) and will, therefore, be dropped from further consideration.

We would now like to relate the impedance functions of the base to the motions of the structure under consideration, Fig. A2. This can be accomplished by noting the following relationships:
\[
\begin{align*}
\begin{bmatrix}
\ddot{x}_b(\omega) \\
\ddot{z}_b(\omega) \\
\ddot{\theta}_b(\omega)
\end{bmatrix}
&= 
\begin{bmatrix}
1 & 0 & z_{cg} \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\ddot{x}(\omega) \\
\ddot{z}(\omega) \\
\ddot{\theta}(\omega)
\end{bmatrix} 
\text{ . . . . . . . Al.} 4
\end{align*}
\]

where \(\ddot{x}(\omega), \ddot{z}(\omega)\) and \(\ddot{\theta}(\omega)\) are displacements of the structure center of gravity and \(z_{cg}\) is the height of the center of gravity above the foundation surface.

We may also write

\[
\begin{align*}
\begin{bmatrix}
\ddot{F}_x(\omega) \\
\ddot{F}_z(\omega) \\
\ddot{F}_\theta(\omega)
\end{bmatrix}
&= 
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
z_{cg} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\ddot{r}_b(\omega) \\
\ddot{r}_b(\omega) \\
\ddot{r}_b(\omega)
\end{bmatrix} 
\text{ . . . . . . . Al.} 5
\end{align*}
\]

Applying Eq. Al.2 (ignoring coupling) and Al.4 to Eq. Al.5, we have

\[
\begin{align*}
\begin{bmatrix}
\ddot{F}_x(\omega) \\
\ddot{F}_z(\omega) \\
\ddot{F}_\theta(\omega)
\end{bmatrix}
&= 
\begin{bmatrix}
\ddot{r}_b(\omega) & 0 & z_{cg} \ddot{r}_b(\omega) \\
0 & \ddot{r}_b(\omega) & 0 \\
z_{cg} \ddot{r}_b(\omega) & 0 & \ddot{r}_b(\omega)
\end{bmatrix}
\begin{bmatrix}
\ddot{x}(\omega) \\
\ddot{z}(\omega) \\
\ddot{\theta}(\omega)
\end{bmatrix} 
\text{ . . . . . . . Al.} 6
\end{align*}
\]

We are now able to describe the structure foundation impedances in terms of the appropriate impedances of the rigid disk.

A2. The Elastic Half-Space Impedance Approximation

Veletzos and Verbic\textsuperscript{13} have presented frequency dependent expressions for the foundation impedances of Eq. Al.6 as follows:
\[ K_{xx}(\omega) = 4.8 \cdot \text{GR} \{1. + i \cdot 0.65 \cdot a_0(\omega)\} \quad \ldots \ldots \quad A2.1 \]

\[ K_{zz}(\omega) = 6.0 \cdot \text{GR} \{1. - \frac{0.224 \cdot a_0^2(\omega)}{1. + 0.64 \cdot a_0^2(\omega)} \]
\[ + i \left(0.75 \cdot a_0(\omega) + \frac{0.179 \cdot a_0^3(\omega)}{1. + 0.64 \cdot a_0^2(\omega)}\right)\} \quad \ldots \ldots \quad A2.2 \]

\[ K_{BB}(\omega) = 4.0 \cdot \text{GR}^3 \{1. - \frac{0.32 \cdot a_0^2(\omega)}{1 + 0.64 \cdot a_0^2(\omega)} + i \left(\frac{0.256 \cdot a_0^3(\omega)}{1 + 0.64 \cdot a_0^2(\omega)}\right)\} \quad \ldots \ldots \quad A2.3 \]

where Poisson's ratio equal to one third (1/3) has been assumed and

\[ G = \text{soil shear modulus of elasticity in the half space.} \]
\[ R = \text{radius of the foundation} \]
\[ a_0(\omega) = \omega R / C_s, \text{ where } C_s \text{ is the shear wave velocity} \]

**A3. Evaluation of Foundation Stiffness for the Prototype Offshore Gravity Structure**

It is necessary to make a number of assumptions in order to evaluate Eqs. A2.1 - A2.3, for the prototype system. We shall simplify these calculations by noting that we will not include additional foundation damping in our model system, therefore, we will not consider these coefficients further.

Shear wave velocity can be expressed as\(^{21}\)

\[ C_s = \sqrt{G/\rho_s} \quad \ldots \ldots \quad A3.1 \]
where, $\rho_s$ - mass density of the soil, we can now write

$$a^2_0(\omega) = (\omega^2 R^2 \rho_s)/G$$

...... A3.2

We will assume that the foundation material in our system has a mean density of

$$\rho_s = 2000 \text{ kg/m}^3$$

and that we are interested in response in the near vicinity of

$$\omega_{\text{mean}} \approx 7.5 \text{ radius/sec.}$$

or,

$$f_{\text{mean}} = 1.2 \text{ Hz.}$$

Finally, we will assume a prototype such that

$$R \approx 40 \text{ meters}$$

Eq. A3.2 becomes

$$a^2_0(\omega) \approx \frac{1.8 \times 10^5}{G}$$

where $G$ is expressed in KN/m$^2$ ($1 \text{ KN} = 1000 \text{ Newton's}$; $1 \text{ KN/m}^2 \approx 21 \text{ lb/ft}^2$).

With these assumptions, we can now write the stiffness portions of Eqs. A2.1 - A2.3 as

$$K_{xx}^b = 192 \ G$$

...... A3.3

$$K_{zz}^b = 240 \ G \left(1 - \frac{4.01 \times 10^4}{G + 1.152 \times 10^5}\right)$$

...... A3.4

$$K_{th}^b = 2.56 \times 10^5 \ G \left(1 - \frac{5.76 \times 10^4}{G + 1.152 \times 10^5}\right)$$

...... A3.5
where translational stiffness values are in KN/m and rotational stiffness is in (KN-m)/radian.

The stiffness relationships of Eqs. A3.3 - A3.5 are plotted in Figs. 3.1 - 3.3 and the equivalent prototype stiffnesses used in this study are indicated. An attempt was made to maintain a consistent relationship between horizontal and rotational stiffness but vertical stiffness was considered to be independent of the other two.

An attempt was made to model three stiffness values which would represent the range of shear modulus change experienced by a dense sand undergoing strong shaking such that shear strain varied from approximately 0.0001 percent to 0.1 percent, as reported by Seed and Idriss. The stiffnesses actually achieved were in this range but somewhat short of the extremes on either end. The actual values were dictated by material availability and space limitations in the model.

The detail of the analysis of model foundation characteristics are contained in APPENDIX B.
FIGURE A1: A GRAVITY STRUCTURE IN AN EXAGGERATED DISPLACED CONFIGURATION