Fig. 5.5 Soil Reaction and Bending Moment Distribution for a Rigid, Free-headed Pile in Cohesive Soil (Broms, 1964a)

Fig. 5.6 Soil Reaction and Bending Moment Distribution for a Rigid, Restrained Pile in Cohesive Soil (Broms, 1964a)
\[ M_{\text{max}} = P_{\text{ult}} (0.5 L + 0.75 D) \] for restrained pile \hspace{1cm} (5.3)

in which: \( M_{\text{max}} \) = maximum pile moment
\( P_{\text{ult}} \) = ultimate lateral load
\( D \) = diameter of pile
\( c_u \) = undrained shear strength of soil
\( L \) = length of embedment of pile

Based on statics considerations on the simplified distribution of maximum soil reactions shown in Fig. 5.5 and 5.6, Broms (1964a) developed a chart for determination of ultimate lateral capacities of rigid piles. This is shown in Fig. 5.7. It should be noted that this graphical relationship applies only for cases where \( M_{\text{max}} \) is less than \( M_{\text{yield}} \), the yield moment of the pile (i.e., the pile behaves as a rigid member).

When a free-headed, flexible pile is loaded laterally, both the pile moments and the soil reactions along the pile will increase. By definition of flexible members, the ultimate lateral capacity of the pile-soil system is determined by the yield moment of the pile shaft. Failure occurs when a failure hinge (a section where the pile material has reached its ultimate strength) is formed somewhere along the pile. In the case of a restrained pile, one or two failure hinges may be formed, depending upon the length of the member. For an intermediate-length pile, the failure hinge will occur at the level of restraint; for a long pile, one failure hinge will be formed at the level of restraint and the other at some depth below the ground surface.

By assuming the soil reactions above the failure hinge to
Fig. 5.7 Ultimate Lateral Resistance of Rigid Pile in Cohesive Soil (Broms, 1964a)
be fully mobilized, Broms (1964a) simplified the soil reaction and bending moment distributions for flexible piles in clays to those shown in Figs. 5.8, 5.9 and 5.10. Dimensionless solutions for these situations are shown in Fig. 5.11. It should be noted from the graph that the ultimate lateral capacity of a flexible pile is independent of the penetration depth and, therefore, there is no point in driving a pile down to a depth greater than that required for the pile to behave as a flexible member (except for cases where greater penetration depths are necessary to support axial loads).

Broms (1964a) has compared the maximum bending moments computed using the above approach with experimental values measured by various investigators and observed good agreement between the two values. Moreover, he pointed out that the calculated maximum bending moment value is not very sensitive to small variations in the assumed soil reaction distribution or the undrained shear strength values.

5.1.2 Piles in Sand

When a free-headed, rigid pile is loaded in the horizontal direction, the pile will rotate as a unit. The soils in front of the pile near the surface will move upwards and those at greater depth will be densified in the direction of pile movements. For a restrained pile, the member will translate horizontally. In both cases, the sands along the back of the pile will be loosened as a result of these pile movements.

Based on load test results, Broms (1964b) suggested that the horizontal soil stress which develops at lateral capacity
Fig. 5.8 Soil Reaction and Bending Moment Distribution for a Flexible, Free-Headed Pile in Cohesive Soil (Broms, 1964a)
Fig. 5.9  Soil Reaction and Bending Moment Distribution for a Flexible, Restrained Pile (Intermediate Length) in Cohesive Soil (Broms, 1964a)

Fig. 5.10  Soil Reaction and Bending Moment Distribution for a Flexible, Restrained Pile (Long Length) in Cohesive Soil (Broms, 1964a)
Fig. 5.11  Ultimate Lateral Resistance of Flexible Pile in Cohesive Soil (Broms, 1964a)
failure is approximately equal to three times the Rankine passive soil stress. The distributions of soil reactions and pile moments at the ultimate stage as proposed by Broms (1964b) are shown in Figs. 5.12 and 5.13. By assuming the maximum soil reactions to be fully mobilized at the ultimate stage and ignoring the active soil stress acting along the back of the pile, Broms (1964b) developed a chart to estimate the ultimate lateral capacities for rigid piles in sands and this is shown in Fig. 5.14. Again, the chart applies only when the maximum bending moment in the pile is less than the yield moment. The maximum moments at the ultimate states shown in Figs. 5.12 and 5.13 are given respectively as:

\[ M_{\text{max}} = P_{\text{ult}} (e + 0.55 \sqrt{\frac{P_{\text{ult}}}{\gamma_s D K_p}}) \text{ for free-headed pile} \quad (5.4) \]

\[ M_{\text{max}} = 0.67 P_{\text{ult}} L \text{ for restrained pile} \quad (5.5) \]

in which: \( \gamma_s \) = submerged unit weight of soil

\( K_p \) = passive soil stress coefficient

Using a similar logic as that for piles in clays, Broms (1964b) proposed soil reactions and pile moment distributions for flexible members in sands and these are shown in Figs. 5.15, 5.16 and 5.17. Dimensionless solutions obtained by Broms (1964b) for this category of laterally loaded piles are shown in Fig. 5.18.

Compared with the measured values of ultimate lateral capacities of piles in sands reported by various investigators, Broms (1964b) noted that the measured values generally exceed
Fig. 5.12  Soil Reaction and Bending Moment Distribution for Rigid, Free-headed Pile in Granular Soil (Broms, 1964b)

Fig. 5.13  Soil Reaction and Bending Moment Distribution for Rigid, Restrained Pile in Granular Soil (Broms, 1964b)
Fig. 5.14  Ultimate Lateral Resistance of Rigid Piles in Granular Soil (Broms, 1964b)
Fig. 5.15 Soil Reaction and Bending Moment Distribution for a Flexible, Free-Headed Pile in Granular Soil (Broms, 1964b)
Fig. 5.16  Soil Reaction and Bending Moment Distribution for a Flexible, Restrained Pile (Intermediate Length) in Granular Soil (Broms, 1964b)

Fig. 5.17  Soil Reaction and Bending Moment Distribution for a Flexible, Restrained Pile (Long Length) in Granular Soil (Broms, 1964b)
Fig. 5.18 Ultimate Lateral Resistance of Flexible Pile in Granular Soil (Broms, 1964b)
the calculated values by approximately 50%. The use of a factor of 3 for the Rankine soil stress at failure conditions is probably too conservative for the analysis.

5.2 Ultimate Capacity of Batter Piles

When large lateral loads are anticipated, it is common practice to use batter piles. Batter piles, sometimes called raking or inclined piles, are piles that are driven at an angle from the vertical. Common pile batters range from 1 horizontal : 12 vertical to 5 horizontal : 12 vertical (Bowles, 1977). It should be noted that installation of batter piles is likely to be more expensive and, in fact, construction limitations may sometimes prohibit the use of a large angle of inclination. These conditions should be thoroughly checked in the design phase.

One effective way of resisting lateral forces from either direction is to have piles battered in opposite directions as shown in Fig. 5.19. As the system is loaded, one pile will be in tension and the other in compression. Both piles will contribute resistance to lateral forces. This type of pile arrangement is commonly used for dolphins.

Simple procedures for determination of capacities of batter piles are not firmly established because of the complexity of this soil-structure interaction problem. Very limited load tests have been performed on batter piles. Results on model tests conducted by Awad and Petrasovits (1968) to determine the effects of pile inclination on load capacity are shown in Fig. 5.20. This can serve as a rough guide for estimation of load
Fig. 5.19  Use of Batter Piles to Resist Lateral Forces
Fig. 5.20 Relative Ultimate Capacity for Different Angle of Inclination (after Awad and Petrasovits, 1968)
capacities of batter piles until more up-to-date information is available.

One major problem associated with the use of batter piles in compressible soil layers has been addressed by Broms (1976). The author pointed out that settlement of soil layers, because of self weight or surcharge load, may impose significant vertical loads along the inclined piles and induce large bending moments that can cause the pile to fail. His idea is further depicted in Fig. 5.21. Because of this, batter piles should not be used or the inclinations should be kept very small in areas where substantial settlements of the soil mass may occur.

5.3 Ultimate Capacity of Pile Groups

The evaluation of the lateral load capacity of a pile group is very complex. Lateral load tests performed on pile groups are rare and they are mainly restricted to small groups of piles. No simple method currently exists to deal with this problem. Some general guidelines will be provided here; they should be used cautiously, with good judgment. If possible, load tests should be performed to verify the computational results.

If the piles behave as rigid members, it will probably be reasonable to adopt an approach similar to that for axial loads. On this basis, the group lateral capacity is given as the smaller of:

1) the sum of lateral load capacities of individual piles,
2) the lateral load capacity of an equivalent single block defined by the exterior piles in the group.
Fig. 5.21 Bending of Batter Piles because of Soil Settlement
The former can be obtained using procedures outlined in Section 5.1. The latter will include the ultimate soil resistance in front of the block plus the shear resistance acting along the two sides and the base of the block (Fig. 5.22).

The failure mode of the soil in the front end of the block (plane surface) will not be identical to that assumed for a circular surface. However, it is likely that the soil resistance acting on a plane surface will be higher than that acting on a curved surface. Therefore, the use of $q_{u}$ (undrained case) and $3K_p\bar{\sigma}_v$ (drained case) as the ultimate soil reactions, as illustrated in Figs. 5.5, 5.6, 5.12 and 5.13, should give an approximate and conservative solution.

If the piles act as flexible members, the failure of the system is caused by yielding of the pile material rather than the soil. In such cases, block failure will be confined to the upper portion of the block located above the failure hinges and the group capacity can be estimated by summation of the lateral load capacities of individual members.

Davisson (1970) pointed out that, under most circumstances, piles behave as individual units if they are spaced more than 8 diameters apart in the direction of the loading and at least 2 1/2 diameters apart in the perpendicular direction.

5.4 Deflections of Vertical Piles

The design of laterally loaded piles may very often be governed by deflections rather than ultimate load capacity. Pile deflections lead to lateral displacements and tilt of the supported structures. This may lower the serviceability of the
Fig. 5.22 Free-Body Diagram of the Group Behavior of a Pile Group under Lateral Loads
structure even though it may be 100% safe from a structural standpoint. The allowable deflection that can be tolerated at working load depends upon the function of the structure. Although large movements for docks or piers are not desirable, the deflections can be relatively large for fender piles or temporary structures.

The evaluation of lateral deflections of horizontally loaded piles has been based on the subgrade reaction model. This stems from a well-known class of problems in structural mechanics known as "beam on elastic foundation." The governing equation for this class of problems is given as:

$$E_p I_p \frac{d^4 y}{dz^4} = -pD$$

(5.6)

in which: $E_p$ = modulus of elasticity of pile material

$I_p$ = moment of inertia of cross-section of pile

$y$ = deflection of pile

$p$ = soil reactive stress

Theoretically, if the magnitude and variation of the reactive stress are known, Equation 5.6 can be solved for deflection at any point along the pile.

Using the above concept, the soil reactions in front of a laterally loaded pile are simulated by a series of horizontal elastic springs (Fig. 5.23). The stiffness of these springs is represented by the horizontal subgrade reaction modulus, $k_h$; the reactive horizontal stress at any point along the pile is related to the amount of deflection at that same point. Studies have shown that this relationship is nonlinear (Davisson
Fig. 5.23 Subgrade Reaction Model for Laterally Loaded Piles
and Prakash, 1963) and the horizontal subgrade reaction modulus for a given magnitude of deflection is given by the slope of the p-y curve at that magnitude of deflection (Fig. 5.24). The p-y curve for any pile-soil system is best derived from full-scale pile load tests and a nonlinear analysis of the problem can be performed by iterative techniques with the help of a computer.

As illustrated by the typical p-y curve shown in Fig. 5.24, the initial portion of the curve can be linearized without sacrificing much accuracy. Under most circumstances, this is valid as long as the load is less than 1/2 to 1/3 of the ultimate load, i.e., under most working load conditions. Assuming the springs to behave in a linear and elastic manner, the reactive horizontal stress at any depth along the pile can be expressed as:

\[ p = -k_n y \]  \hspace{1cm} (5.7)

in which: \( k_n \) = horizontal subgrade reaction modulus

The solution of Equations 5.6 and 5.7 requires a knowledge of the magnitude and variation of \( k_n \) with depth. These can be inferred through load test results on instrumented piles. A typical plot of \( k_n \) versus depth for overconsolidated clays is shown in Fig. 5.25. Granular soils and normally consolidated cohesive soils, on the other hand, exhibit a variation as shown in Fig. 5.26. For simplicity, a constant modulus is often used for the case of overconsolidated clays and a linearly increasing modulus with depth is assumed for normally
Fig. 5.24  Typical p-y Curve
Fig. 5.25  Variation of $k_h$ with Depth for Over-consolidated Clay (after Davison, 1970)

Fig. 5.26  Variation of $k_h$ with Depth for Normally Consolidated Clay and Granular Soil (after Davison, 1970)
consolidated cohesive soils and granular materials (Davisson, 1970).

Analytical closed-form solutions are readily available for the two conditions described below. For the case of a linearly increasing modulus, the stiffness of the "springs" is represented by a new constant parameter, \( n_h \), which is related to \( k_h \) by:

\[
k_h = n_h \frac{z}{D}
\]  

(5.8)

in which: \( n_h = \) coefficient of horizontal subgrade modulus variation

\( z = \) depth below the ground surface

The two parameters, \( k_h \) for constant modulus and \( n_h \) for linearly increasing modulus, as suggested by various investigators, are shown in Tables 5.2 and 5.3.

By adopting the above concept, Broms (1964) developed charts to give the deflection of piles at ground surface under working load conditions. The charts for the two cases where \( k_h \) is constant with depth and is increasing linearly with depth are shown in Figs. 5.27 and 5.28, respectively. The use of these charts requires a knowledge of the dimensionless parameters, \( \beta \) and \( n \), defined as:

\[
\beta = \frac{\sqrt{k_h D/4E_p}}{p} 
\]  

(5.9)

\[
n = \frac{\sqrt{n_h/E_p}}{p} 
\]  

(5.10)

In cases where repeated loadings can occur, Davisson (1970) suggested that \( k_h \) (or \( n_h \)) be reduced to 30% of the value used for sustained loadings. He attributed this to degradation of
<table>
<thead>
<tr>
<th>Type of Clay</th>
<th>Stiff</th>
<th>Very Stiff</th>
<th>Hard</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_u$ lb/ft$^2$ kN/m$^2$</td>
<td>2000 - 4000</td>
<td>4000 - 8000</td>
<td>&gt; 8000</td>
</tr>
<tr>
<td></td>
<td>96 - 192</td>
<td>192 - 383</td>
<td>&gt; 383</td>
</tr>
<tr>
<td>$k_h$ lb/ft$^2$ kN/m$^2$</td>
<td>100,000</td>
<td>200,000</td>
<td>400,000</td>
</tr>
<tr>
<td></td>
<td>15,700</td>
<td>31,400</td>
<td>62,800</td>
</tr>
</tbody>
</table>

Table 5.2 Values of Horizontal Subgrade Modulus for Overconsolidated Clay (Terzaghi, 1955)
For Sands:

<table>
<thead>
<tr>
<th>Source</th>
<th>Loose Dry</th>
<th>Loose Submerged</th>
<th>Medium Dry</th>
<th>Medium Submerged</th>
<th>Dense Dry</th>
<th>Dense Submerged</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terzaghi (1955)</td>
<td>14000</td>
<td>8000</td>
<td>42000</td>
<td>28000</td>
<td>112000</td>
<td>68000</td>
<td>lb/ft³</td>
</tr>
<tr>
<td></td>
<td>2200</td>
<td>1300</td>
<td>6600</td>
<td>4400</td>
<td>17600</td>
<td>10700</td>
<td>KN/m³</td>
</tr>
<tr>
<td>Howe (1956)</td>
<td>5000</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>160000</td>
<td>-</td>
<td>lb/ft³</td>
</tr>
<tr>
<td></td>
<td>790</td>
<td>1300</td>
<td>25100</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>KN/m³</td>
</tr>
<tr>
<td>Reese, et al.</td>
<td>-</td>
<td>35000</td>
<td>-</td>
<td>104000</td>
<td>-</td>
<td>216000</td>
<td>lb/ft³</td>
</tr>
<tr>
<td>(1974)</td>
<td></td>
<td>5500</td>
<td>-</td>
<td>16300</td>
<td>-</td>
<td>33900</td>
<td>KN/m³</td>
</tr>
</tbody>
</table>

For Normally Loaded Soils: (Davisson, 1970)

Organic Silt, 700 - 5000 lb/ft³ (110 - 790 KN/m³)

Peat, 350 lb/ft³ (55 KN/m³)

Clay, 67 c_u/D lb/ft³ (11 c_u/D KN/m³)

Table 5.3 Values of Coefficient of Modulus Variation
Fig. 5.27 Lateral Deflection at Ground Surface - Subgrade Modulus Constant with Depth (Broms, 1964a)

Fig. 5.28 Lateral Deflection at Ground Surface - Subgrade Modulus Increasing Linearly with Depth (Broms, 1964b)
soil reactions caused by repeated loadings.

When the subgrade reaction model is applied to predict group movements, a reduction in \( k_h \) (or \( n_h \)) should also be used, as shown in Table 5.4.

5.5 **Summary**

The design of laterally loaded piles is one of the more complicated aspects of pile foundations. Very few load tests have been reported in the literature. Most of the recent work is based on computer techniques. Nevertheless, hand calculation methods can provide a rapid and rough estimate of the solution.

It has been pointed out that the lateral capacity of the pile-soil system may be limited by the strength of either the pile or the soil. The former is a flexible pile and the latter is a rigid pile. A designer should understand the fundamental differences in behavior and application between these two types of piles to achieve a proper design. The degree of end restraint may also significantly influence the lateral capacity of the system and therefore should be thoroughly examined.

Batter piles are commonly used to resist large lateral forces. However, construction limitations or field conditions may very often prohibit the use of a large angle of inclination. The occurrence of any potential problem should be investigated in the design phase. Guidelines are provided to estimate the capacities of batter piles. These are based on very limited model test results and should be used cautiously.
<table>
<thead>
<tr>
<th>Pile Spacing in the Direction of Loading</th>
<th>Effective k Values for Pile Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D</td>
<td>1.00 (k_h)</td>
</tr>
<tr>
<td>7D</td>
<td>0.85 (k_h)</td>
</tr>
<tr>
<td>6D</td>
<td>0.70 (k_h)</td>
</tr>
<tr>
<td>5D</td>
<td>0.55 (k_h)</td>
</tr>
<tr>
<td>4D</td>
<td>0.40 (k_h)</td>
</tr>
<tr>
<td>3D</td>
<td>0.25 (k_h)</td>
</tr>
</tbody>
</table>

Note: Pile spacing normal to the direction of loading has no influence if it is greater than 2.5 D.

Table 5.4  Subgrade Reaction Modulus of Pile Groups (after Davisson, 1970)
The best approach to analyze a laterally loaded pile group is by pile load test. If this is not economically feasible, the approximate procedures outlined in Section 5.3 can probably be used.

Similar to piles under compressive loads, piles subjected to lateral forces must also conform to deformation requirements, as well as ultimate capacity. The method presented herein to predict the amount of deflection upon application of loads is based on the subgrade reaction theory. Although this theory has some inherent limitations (e.g., fails to recognize the continuity of soils), it is widely used in practice and can be adopted under working load conditions.

In situations where the lateral loads are significant and batter piles cannot or should not be used, various methods suggested by Broms (1976) can be employed to increase the lateral resistance of vertical piles. These are shown in Fig. 5.29. Most of these methods are intended to increase the stiffness of the piles near the ground surface.
Fig. 5.29  Methods to Increase the Lateral Resistance of Vertical Piles (Broms, 1976)
CHAPTER 6

OTHER CONSIDERATIONS

In addition to the load carrying capacities, the design of pile foundations often incorporates factors that form a major portion of the design considerations. Some of the more common of these will be described in this chapter. The degree of importance of these factors depends upon the foundations, the site conditions and the applied loads. Other considerations may also influence the design of pile foundations at a given site; these have to be left to the ingenuity of the designer since it is impossible to discuss every possibility in this report.

6.1 Scour Around Pile Foundations

Scour can be defined as a process which involves the removal of granular material on the sediment bed. It occurs around any obstacle that obstructs the normal water-flow patterns. This erosive action around pile foundations has always been a concern for the design of coastal structures. The formation of a scour pit (depression surrounding an obstruction) as a result of this erosion reduces the lateral capacity of the pile. Moreover, the unsupported length of the pile will increase, thus reducing the frictional resistance of the pile under axial loadings. If not properly taken into account in the design phase, this phenomenon may markedly reduce the life of the structure.

Scour may result from currents, waves, ship motion or rise
of water level. The factors that are likely to control are currents and waves. Current and wave-induced scour are dominated by parameters such as velocity of flow, wave characteristics, water depth, pile diameter and the geological history of the site. Scour that occurs as a result of rise of water level is important at sites where large floods occur frequently, especially if the rise occurs in a very short period of time.

Earlier methods to predict scour depth were largely based on experience and a large number of parameters related to the problem were ignored. Kuhn and Williams (1961) suggested the scour depth to be the depth where there is a sudden increase in penetration resistance. While this may be adequate to locate the depth of the loose deposit layer, the scour depth can have a large deviation, depending upon the current velocity and the dimensions of the obstacles. Terzaghi and Peck (1967) proposed the maximum scour depth for design to be four times the amount of water level rise anticipated. This proposal is impractical in places where there is a large fluctuation (e.g., 8 ft) in water level.

Later investigations on the problem were mainly conducted in laboratory flumes. The primary objective of these investigations is to establish criteria and guidelines by which designers can assess the depth of scour under certain given conditions. Important factors are identified with varying degrees of success. However, the results obtained are qualitative rather than quantitative.

Palmer (1969) observed oscillatory wave-induced scour and
developed a schematic view of the general hydrodynamics in the vicinity of an obstruction (Fig. 6.1). He considered the pattern of secondary flow or turbulence to be a major factor in the removal of granular material around an obstacle. He suggested the main scouring force to be the primary vortex that develops in front of the cylinder.

Laursen (1962) studied the phenomenon of scour in a laboratory flume and demonstrated that there is an equilibrium or limiting depth of scour for any given set of conditions. He also pointed out that the depth of scour for a group of piles does not depend upon the proximity of adjacent piles unless the scour pits overlapped, in which case the depth of scour can be estimated by the solution for contraction of a river channel. This will probably accelerate the formation of scour pits. Similar investigations by Palmer (1969) showed that the pit diameter is proportional to the diameter of the pile, but is largely independent of surge velocity. He further noted that scour pits caused by inclined piles are of equal dimensions to those caused by identical vertical piles.

Machemehl and Abad (1975) investigated scour phenomenon caused by the combined effect of oscillatory waves and unidirectional current around a model cylindrical pile foundation. They studied the effects of current velocity, wave characteristics and diameter of piles on extent of scour. They concluded that:

1) the increase of water particle velocity adjacent to the pile is the main scouring mechanism.
Fig. 6.1  Mechanism of Scour (after Palmer, 1969)
2) the addition of oscillatory waves in a unidirectional flow field increases the rate of scour development.

3) waves with long wavelength produce scour pits at a faster rate as compared with that produced by waves of short wave length.

4) scour effect is directly proportional to the diameter of the pile.

Since the severity of scour is very site-specific, the best approach to estimate the scour pit dimensions is to consult professionals with local experience. If this is not possible, a large margin of safety should be used. Allowances for this effect should be made in the selection of pile embedment so that adequate penetration will remain after scour.

6.2 Buckling of Fully and Partially Embedded Piles

It is well known that only very short members can be stressed to their yield point under compression; the usual situation is that buckling, or sudden bending as a result of instability, occurs prior to development of full material strength of the member. The buckling strength of a pile depends upon the end conditions, the properties of the pile, and the amount of horizontal support.

Early studies of the buckling of piles showed that the occurrence of buckling for fully embedded piles is a remote possibility, even in soft soils. Under most circumstances, minimal horizontal support will be able to prevent the piles from buckling under compressive forces. Bjerrum (1957) has shown that buckling of fully embedded piles in a soil with a constant
modulus need be considered only in cases where:

\[ \frac{I_p}{A^2} = \frac{\sigma_{\text{max}}}{4k_h DE_p} \]  

(6.1)

in which:  
- \( I_p \) = moment of inertia for weak axis bending  
- \( A \) = cross-sectional area of pile  
- \( \sigma_{\text{max}} \) = yield stress of pile material  
- \( k_h \) = subgrade reaction modulus  
- \( D \) = diameter of pile  
- \( E_p \) = modulus of elasticity of pile material

In general, the above restraint can be satisfied except for very slender piles in soft soils.

Davissson (1963) presented theoretical solutions to evaluate the buckling potential of fully embedded piles for various boundary conditions shown in Fig. 6.2. These boundary conditions represent idealized situations for design. For total fixity at the top, the deck or cap must be rigidly attached to the piles; for total fixity at the bottom, the soil must be a firm material into which the piles are driven a considerable depth. Whether or not the ends can translate depends largely upon the structural configuration. Horizontal stability can be provided by diagonal bracing, shear walls, adjacent structures, pile caps, etc. For most real situations, the degree of rotational and translational restraints do not exist at these extremes. However, analyses made can provide bounding solutions for designers. An appropriate choice of end conditions is based on the judgment of the designer.
Fig. 6.2  Different Boundary Conditions for Piles
(after Davisson, 1963)
For the case of constant subgrade reaction modulus with depth, Davisson (1963) derived the following equation to estimate the elastic pile buckling loads for fully embedded piles:

\[ Q_{cr} = U_{cr} \frac{E_p l_p}{R^2} \]  

(6.2)

in which: \( Q_{cr} \) = critical buckling load
\( U_{cr} \) = dimensionless buckling parameter for fully embedded piles in soils with constant modulus
\( R = \sqrt{E_p l_p/k} = \sqrt{E_p l_p/k} \)

The dimensionless parameter, \( U_{cr} \), can be obtained from Fig. 6.3 and is a function of both the end conditions and the dimensionless length parameter, \( \lambda_{max} \), defined as:

\[ \lambda_{max} = \frac{L}{R} \]  

(6.3)

in which: \( \lambda_{max} \) = dimensionless length parameter for fully embedded piles in soils with constant modulus
\( L \) = length of embedment of pile

It can be seen in Fig. 6.3 that the pile boundary conditions can have a strong influence on the critical buckling load. The lowest value of \( Q_{cr} \) is associated with piles that have no restraint with both the upper and lower ends.

With a linearly increasing subgrade reaction modulus, Davisson (1963) obtained the following equation to compute the critical buckling load for fully embedded piles:

\[ Q_{cr} = V_{cr} \frac{E_p l_p}{T^2} \]  

(6.4)

in which: \( V_{cr} \) = dimensionless buckling parameter for fully em-
Fig. 6.3  Buckling Load Versus Length for Constant Modulus (Davisson, 1963)
bedded piles in soils with linearly increasing modulus

\[ T = \sqrt{\frac{E_p I_p}{n_h}} \]

The dimensionless buckling load parameter, \( V_{cr} \), is shown in Fig. 6.4 as a function of both the end conditions and the dimensionless length parameter, \( z_{max} \), defined as:

\[ z_{max} = \frac{L}{T} \]  \hspace{1cm} (6.5)

in which: \( z_{max} \) = dimensionless length parameter for fully embedded piles in soils with linearly increasing modulus

Partially embedded piles that extend through water are commonly used for coastal structures such as docks and piers. The buckling potential of these piles is larger than that of fully embedded piles. An approximate procedure to deal with this problem was developed by Davison and Robinson (1965). The buckling load is estimated through an analysis of an "equivalent" system under which the pile is assumed to be free-standing with a fixed base located at some distance below the ground surface. The distance from the ground surface to the point of fixity is directly related to the strength of the soil. The lower the strength value, the deeper will be the point of fixity. This concept is further illustrated in Fig. 6.5.

The critical buckling load for the "equivalent" system shown in Fig. 6.5 can be computed from the following equations:

\[ Q_{cr} = \frac{\pi^4 E_p I_p}{4(L_d + L_s)^2} = \frac{\pi^4 E_p I_p}{4L_e^2} \]  \hspace{1cm} (6.6)
Fig. 6.4  Buckling Load Versus Length for Linearly Increasing Modulus with Depth (Davisson, 1963)
Fig. 6.5 Point of Fixity Method
\[ Q_{cr} = \frac{\pi^2 E_P I_P}{(L_u + L_s)^2} = \frac{\pi^2 E_P I_D}{L_e^2} \] for fixed-translating head (6.7) condition

in which: \( L_u \) = length of unsupported portion of pile
\( L_s \) = equivalent length of embedded portion of pile
\( L_e \) = equivalent length of the pile

Therefore, the problem lies essentially in the determination of \( L_s \).

Davisson and Robinson (1965) presented the solution in non-dimensional form. In the case of a constant modulus with depth, Fig. 6.6 can be used to obtain \( L_s \). In the figure, \( J_R \) and \( S_R \) are given respectively as:

\[ J_R = \frac{L_u}{R} \] (6.8)
\[ S_R = \frac{L_s}{R} \] (6.9)

in which: \( J_R \) = dimensionless parameter for unsupported portion of piles in soils with constant modulus
\( S_R \) = dimensionless parameter for embedded portion of piles in soils with constant modulus

Once the equivalent length is known, the buckling load can be estimated from Equations 6.6 and 6.7.

In the case of a linearly increasing modulus with depth, Fig. 6.7 can be used. The parameters in the plot are defined as follows:

\[ J_T = \frac{L_u}{T} \] (6.10)
\[ S_T = \frac{L_s}{T} \] (6.11)

in which: \( J_T \) = dimensionless parameter for unsupported portion
Fig. 6.6  Depth of Fixity for Buckling in Soils with Constant Subgrade Modulus (Davisson and Robinson, 1965)

Fig. 6.7  Depth of Fixity for Buckling in Soils with Linearly Increasing Modulus with Depth (Davisson and Robinson, 1965)
of piles in soils with linearly increasing modulus

\( S_T = \text{dimensionless parameter for embedded portion of piles in soils with linearly increasing modulus} \)

Again, the critical buckling load can be estimated from Equations 6.6 and 6.7.

Although the method appears to be sound, the point of fixity method does not have any theoretical justification and in fact it violates the principles of equilibrium and compatibility. Nevertheless, Davison and Robinson (1965) stated that for most practical purposes, the replacement of a partially embedded pile by an equivalent fixed base pile with no horizontal support can be used as long as \( z_{\text{max}} \) is greater than 4 (soil modulus constant with depth) or \( z_{\text{max}} \) is greater than 4 (soil modulus linearly increasing with depth). According to the investigators, these requirements can be met under most practical situations.

6.3 Axial Capacity of Initially Bent Piles

Piles are often bent during driving, particularly when long slender piles are driven into soils that contain rock fragments. This results in reduced capacity under compressive load from that predicted for a concentric load on a straight pile. Failure of a bent pile under compressive load will occur when either the maximum bending stress in the pile reaches the yield strength of the pile material or the maximum soil reaction reaches the yield strength of the soil surrounding the pile. The tolerance level for this type of loading condition depends upon three fac-